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A NOTE ON THE NEW KUMARASWAMY ALPHA POWER INVERTED EXPONENTIAL FAMILY OF C.D.F.

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ABSTRACT: The corresponding cdf of the new Kumaraswamy alpha power inverted exponential distribution is expressed as [1];

$$M(t) = 1 - \left(1 - \left(\frac{\alpha^{e^{-\frac{\lambda}{t}}} - 1}{\alpha - 1}\right)^{\psi}\right)^{b},$$

where $\psi > 0$, $b > 0$, $\lambda > 0$ and $\alpha \in R^{+} - \{1\}$.

Also of interest to the specialists is the task of approximating the Heaviside function

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases}$$

where t_0 is the "median" with the new cumulative function in the Hausdorff sense.

Numerical examples on real datasets (1. *data_Storm_IDs* [9]; 2. *data_Epicenter* [10]; 3. *data_Communication* [11]; 4. *data_Leukaemia* [10]) using *CAS Mathematica*, illustrating our results are given.

AMS Subject Classification: 41A46

Key Words: new Kumaraswamy alpha power inverted exponential c.d.f. (KAPIEcdf), Heaviside step–function $h_{t_0}(t)$, Hausdorff distance

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1. INTRODUCTION AND PRELIMINARIES

Definition 1. *In [1] the authors consider an extension of the Kumaraswamy alpha power inverted exponential c.d.f., which can be applied to life time data:*

$$M(t) = 1 - \left(1 - \left(\frac{\alpha^{e^{-\frac{\lambda}{t}}} - 1}{\alpha - 1}\right)^{\psi}\right)^{b}, \tag{1}$$

where $\psi > 0$, $b > 0$, $\lambda > 0$ *and* $\alpha \in R^{+} - \{1\}$, $t > 0$.

Definition 2. *The shifted Heaviside step function is defined by*

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases} \tag{2}$$

Definition 3. *[2] The Hausdorff distance (the H–distance)* $\rho(f,g)$ *between two interval functions* f, g *on* $\Omega \subseteq \mathbb{R}$, *is the distance between their completed graphs* $F(f)$ *and* $F(g)$ *considered as closed subsets of* $\Omega \times \mathbb{R}$. *More precisely,*

$$\rho(f,g) = \max\{\sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\},$$

wherein $||.||$ *is any norm in* $\mathbb{R}^2$, *e. g. the maximum norm* $||(t,x)|| = \max\{|t|,|x|\}$; *hence the distance between the points* $A = (t_A, x_A)$, $B = (t_B, x_B)$ *in* $\mathbb{R}^2$ *is* $||A - B|| = max(|t_A - t_B|, |x_A - x_B|)$.

For some generalized family of distributions, see [3]–[8].

In this note we study the Hausdorff approximation of the Heaviside function $h_{t_0}(t)$ by the family $M(t)$.

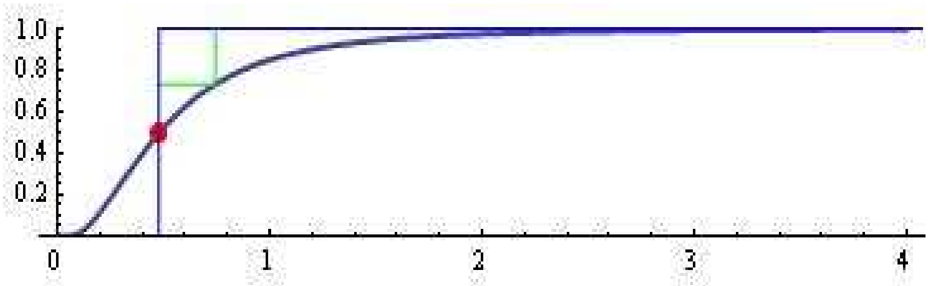

Figure 1: The model (1) for $b = 4.3$, $\alpha = 30.4$, $\lambda = 0.3$, $\psi = 1.1$ and $t_0 = 0.479008$; H–distance $d = 0.265683$.

2. MAIN RESULTS

2.1. A NOTE ON THE NEW KUMARASWAMY ALPHA POWER INVERTED EXPONENTIAL CDF (KAPIECDF) (1)

The investigation of the characteristic "supersaturation" of the model (1) to the horizontal asymptote is important.

Sensitive analysis for the "saturation in the Hausdorff sense".

Let t_0 is the value for which $M(t_0) = \frac{1}{2}$.

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the (cdf) $M(t)$ (1) satisfies the relation

$$M(t_0 + d) = 1 - d. \tag{3}$$

For given α, λ, ψ, b and t_0, the nonlinear equation (3) has unique positive root – d.

The model (1) for $b = 4.3$, $\alpha = 30.4$, $\lambda = 0.3$, $\psi = 1.1$ and $t_0 = 0.479008$ is visualized on Fig. 1.

From the nonlinear equation (3) we have: $d = 0.265683$.

The model (1) for $b = 5.9$, $\alpha = 40$, $\lambda = 0.15\,\psi = 0.64$ and $t_0 = 0.0896254$ is visualized on Fig. 2.

From the nonlinear equation (3) we have: $d = 0.123451$.

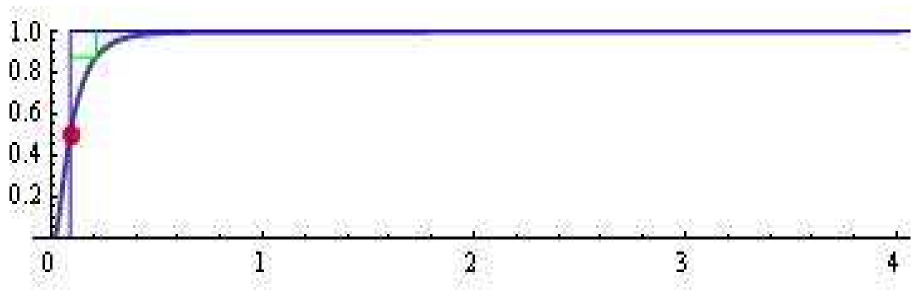

Figure 2: The model (1) for $b = 5.9$, $\alpha = 40$, $\lambda = 0.15$, $\psi = 0.6$ and $t_0 = 0.0896254$; H–distance $d = 0.123451$.

2.2. APPLICATIONS

Example 1. Storm worm one of the most biggest cyber threats of 2008.

We analyze the following data [9]

$$data\_Storm\_IDs := \{\{1, 0.843\}, \{4, 0.926\}, \{5, 0.954\}, \{6, 0.967\}, \{7, 0.976\}, \{8, 0.981\}, \{9, 0.985\}, \{10, 0.991\}, \{22, 0.995\}, \{38, 0.997\}, \{51, 0.998\}, \{64, 0.9985\}, \{74, 0.999\}, \{83, 1\}, \{100, 1\}, \{367, 1\}\}$$

The cdf $M(t)$ for $\alpha = 5.1$, $\psi = 1.9$, $\lambda = 0.5$, $b = 3.63637$ is visualized on Fig. 3.

Example 2. We analyze the following data [10]

$$data\_Epicenter := \{\{39, 0\}, \{59, 0.026\}, \{60, 0.034\}, \{62, 0.064\}, \{66, 0.083\}, \{69, 0.119\}, \{74, 0.153\}, \{77, 0.153\}, \{81, 0.186\}, \{84, 0.218\}, \{89, 0.234\}, \{98, 0.260\}, \{102, 0.301\}, \{107, 0.349\}, \{119, 0.385\}, \{123, 0.413\}, \{128, 0.446\}, \{142, 0.468\}, \{158, 0.551\}, \{164, 0.571\}, \{176, 0.654\}, \{183, 0.750\}, \{193, 0.837\}, \{199, 0.853\}, \{212, 0.885\}, \{217, 0.901\}, \{222, 0.913\}, \{226, 0.933\}, \{229, 0.949\}, \{233, 0.965\}, \{236, 0.984\}, \{238, 1.000\}\};$$

The cdf $M(t)$ for $\alpha = 10.05$, $\psi = 27$, $\lambda = 5.9$, $b = 13.1457$ is visualized on Fig. 4.

Example 3. We analyze the following data [11]

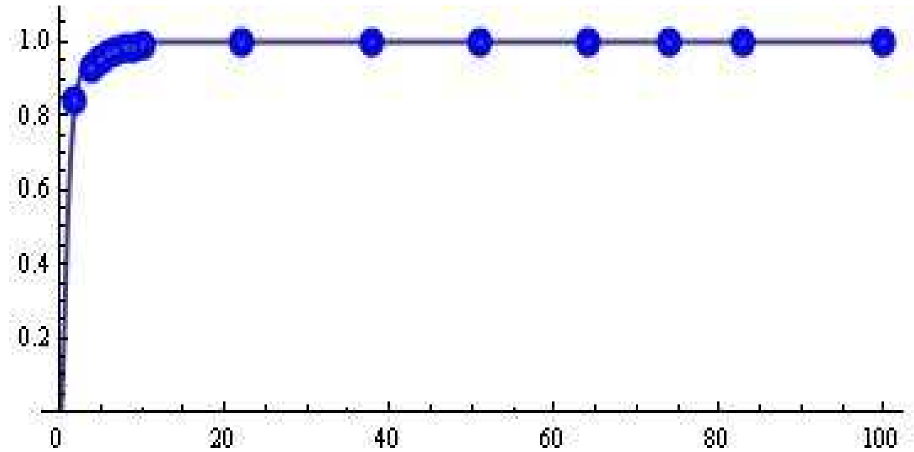

Figure 3: The fitted model $M(t)$.

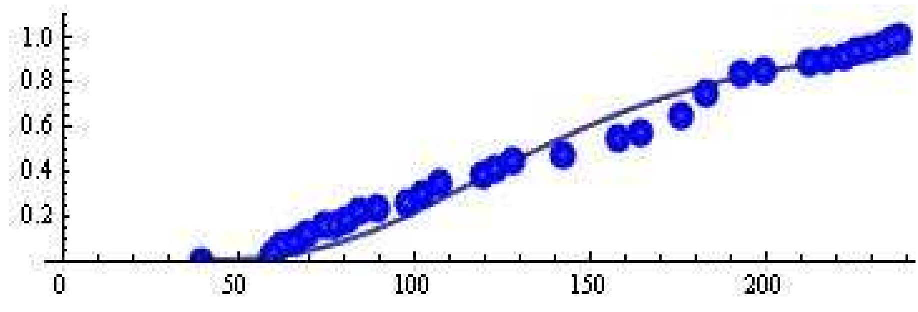

Figure 4: The fitted model $M(t)$.

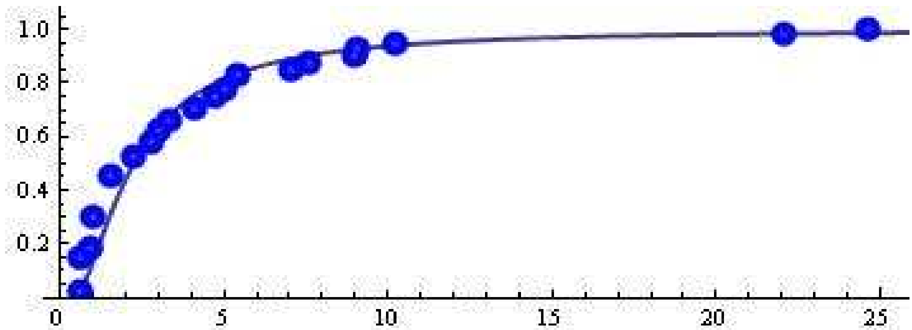

Figure 5: The fitted model $M(t)$.

$$data\_Communication := \{\{0.584, 0.027\}, \{0.649, 0.147\}, \{0.909, 0.187\},$$
$$\{1.039, 0.303\}, \{1.558, 0.453\}, \{2.208, 0.527\}, \{2.792, 0.580\},$$
$$\{3.052, 0.627\}, \{3.312, 0.657\}, \{4.091, 0.707\}, \{4.740, 0.753\},$$
$$\{5, 0.780\}, \{5.390, 0.827\}, \{7.078, 0.853\}, \{7.597, 0.877\},$$
$$\{8.961, 0.903\}, \{9.091, 0.927\}, \{10.195, 0.950\}, \{22.078, 0.980\}, \{24.610, 1\}\};$$

The cdf $M(t)$ for $\alpha = 4.1$, $\psi = 10$, $\lambda = 0.15$, $b = 1.97705$ is visualized on Fig. 5.

Example 4. We analyze the following data [10]

$$data\_Leukaemia := \{\{1, 0.086\}, \{3, 0.184\}, \{4, 0.303\}, \{5, 0.336\},$$
$$\{7, 0.368\}, \{8, 0.401\}, \{16, 0.461\}, \{17, 0.493\}, \{22, 0.546\},$$
$$\{26, 0.576\}, \{30, 0.609\}, \{39, 0.638\}, \{43, 0.671\}, \{56, 0.734\},$$
$$\{64, 0.822\}, \{100, 0.849\}, \{108, 0.882\}, \{121, 0.911\}, \{134, 0.941\},$$
$$\{143, 0.970\}, \{156, 1.000\}\};$$

The cdf $M(t)$ for $\alpha = 3$, $\psi = 2.2267$, $\lambda = 1.169$, $b = 0.550582$ is visualized on Fig. 6.

From the Fig. 3 and 5, it can be seen that the "supersaturation" by the (cdf) $M(t)$ is faster.

Obviously, the new model (1) can also be used for approximating of some "specific data".

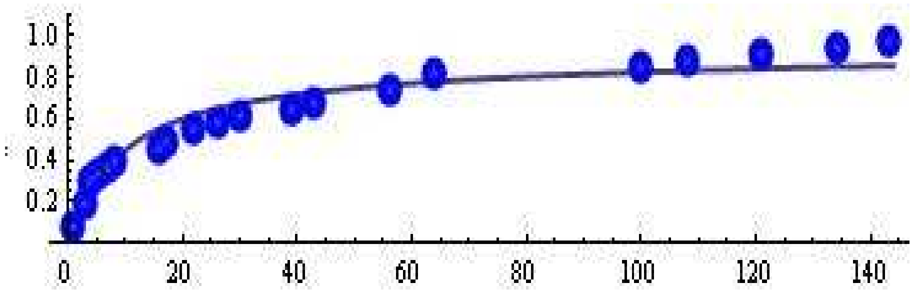

Figure 6: The fitted model $M(t)$.

In concrete case, the conducted analysis in the article shows that the authors of [1] are right when they claim that (KAPIE) distribution produces very good results when ”specific data” are fitted.

For other approximation and modelling results, see [12]–[29].

We hope that the results will be useful for specialists in this scientific area.

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