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ON A MODIFICATIONS OF THE TRUNCATED CAUCHY POWER WEIBULL AND ARCSINE EXPONENTIATED WEIBULL MODELS. SOME APPLICATIONS

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ABSTRACT: In this paper we study some intrinsic properties of the truncated Cauchy power Weibull model [1] and Arcsine exponentiated Weibull model [8]. The models have a certain right to exist in the treatment of issues from different fields of scientific knowledge. We also study the "saturation" in the Hausdorff sense. Some modifications with "polynomial variable transfer" are also given. Numerical examples are presented using *CAS MATHEMATICA*.

AMS Subject Classification: 41A46
Key Words: Truncated Cauchy power Weibull model, Arcsine exponentiated Weibull model, "Saturation" in the Hausdorff sense, Models with "polynomial variable transfer"

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1. INTRODUCTION

In [1] the authors proposed the following new truncated Cauchy power Weibull distribution with cdf:

$$M(t;\alpha,\lambda,\theta) = \frac{4}{\pi}\arctan\left(\left(1-e^{-\lambda t^{\theta}}\right)^{\alpha}\right) \quad (1)$$

for $\alpha > 0$, $\lambda > 0$, $\theta > 0$.

In [2] Bantan, Jamal, Chesneau and Elgarhy introduced a new G–Family of distribution with c.d.f.

$$H(t) = e^{\alpha_1\beta_1\left(1-\frac{1}{G(t)}\right)}\left(2 - e^{\beta_1\left(1-\frac{1}{G(t)}\right)}\right)^{\alpha_1} \tag{2}$$

where $\alpha_1, \beta_1 \in R^+$ and $G(t)$ is a c.d.f. of a baseline continuous distribution.

The following result shows some inequalities involving $H(t)$ (see, Proposition 1 [2]):

$$e^{\alpha_1\beta_1\left(1-\frac{1}{G(t)}\right)}\left(2 - G(t)^{\beta_1}\right)^{\alpha_1} \le H(t) \le 2^{\alpha_1} e^{\alpha_1\beta_1\left(1-\frac{1}{G(t)}\right)}. \tag{3}$$

Definition 1. Formally, we define the following corresponding c.d.f.:

$$M_1(t) = H(t); \quad G(t) = M(t). \tag{4}$$

Definition 2. The *shifted Heaviside step function* is defined by

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases}$$

Definition 3. The Hausdorff distance [3] (the H–distance) $\rho(f,g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{\sup_{A\in F(f)} \inf_{B\in F(g)} ||A-B||, \sup_{B\in F(g)} \inf_{A\in F(f)} ||A-B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

Definition 4. We define the following new model with "polynomial variable transfer":

$$\begin{aligned} M_2(t) &= A\tfrac{4}{\pi}\arctan\left(1 - e^{-R(t)}\right), \\ R(t) &= \sum_{i=0}^{n} a_i t^i; \;\; a_0 = 0. \end{aligned} \tag{5}$$

Definition 5. In [8] He, Ahmad, Afify and Goual examine the following three–parameter arcsine exponential – Weibull (ASE–W) distribution with cdf:

$$M_3(t;\alpha,\lambda,\theta) = \frac{2}{\pi}\arcsin\left(\left(1 - e^{-\lambda t^\theta}\right)^\alpha\right) \tag{6}$$

for $\alpha > 0$, $\lambda > 0$, $\theta > 0$.

Definition 6. Following the ideas given in [2] for generating a new family we formally, define the following corresponding c.d.f.:

$$M_4(t) = H(t); \quad G(t) = M_3(t). \tag{7}$$

Definition 7. We define the following new model with "polynomial variable transfer":

$$\begin{aligned} M_5(t) &= A\tfrac{2}{\pi} \arcsin\left(1 - e^{-R(t)}\right), \\ R(t) &= \sum_{i=0}^{n} a_i t^i; \;\; a_0 = 0. \end{aligned} \tag{8}$$

In this note we study some properties of the new families.

2. MAIN RESULTS. NUMERICAL EXAMPLES

2.1. THE MODEL $M_1(T)$ (4)

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the sigmoid - (4) satisfies the relation

$$M_1(t_0 + d) = 1 - d. \tag{9}$$

When studying the intrinsic properties of the $M_1(t)$, it is also appropriate to study the "saturation" to the horizontal asymptote.

The model (4) and two–sided estimations (3) with $G(t) = M(t)$ for fixed $\alpha_1 = 0.08$, $\beta_1 = 0.2999$ and $\theta = 1.1$; $\lambda = 1.5$; $\alpha = 0.7$, $t_0 = 0.00557798$; H–distance $d = 0.0561585$ is plotted on Fig. 1.

Studies of "saturation" in the Hausdorff sense at the "median" level show that the new model has its worthy place among the "family of models" designed to analyze and approximate "specific" cumulative data.

Example 1. For example we analyze the specific data [4].

The fitted model is depicted on Fig. 2.

Example 2. Storm worm one of the most biggest cyber threats of 2008.

We analyze the following data [5]

$$\begin{aligned} &data\_Storm\_IDs := \{\{1.8, 0.843\}, \{4, 0.926\}, \{5, 0.954\}, \{6, 0.967\}, \\ &\{7, 0.976\}, \{8, 0.981\}, \{9, 0.985\}, \{10, 0.991\}, \{22, 0.995\}, \\ &\{38, 0.997\}, \{51, 0.998\}, \{64, 0.9985\}, \{74, 0.999\}, \{83, 1\}, \{100, 1\}\} \end{aligned}$$

The fitted model (4) is visualized on Fig. 3.

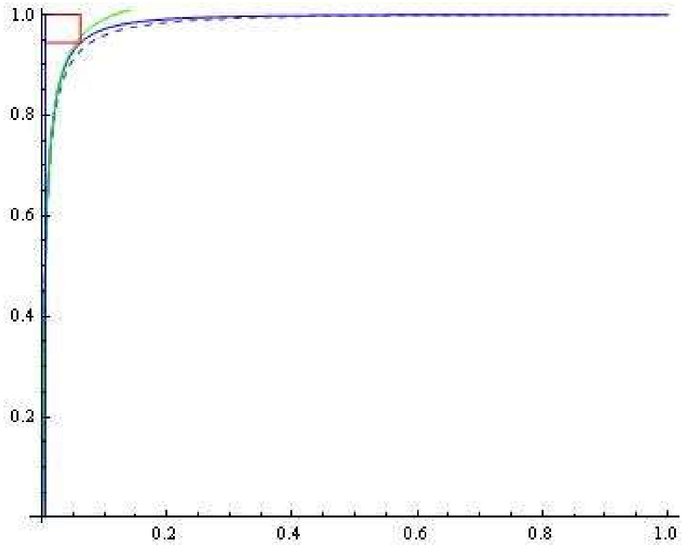

Figure 1: The model (4) and two–sided estimations (3) with $G(t) = M(t)$ for fixed $\alpha_1 = 0.08$, $\beta_1 = 0.2999$ and $\theta = 1.1$; $\lambda = 1.5$; $\alpha = 0.7$, $t_0 = 0.00557798$; H–distance $d = 0.0561585$.

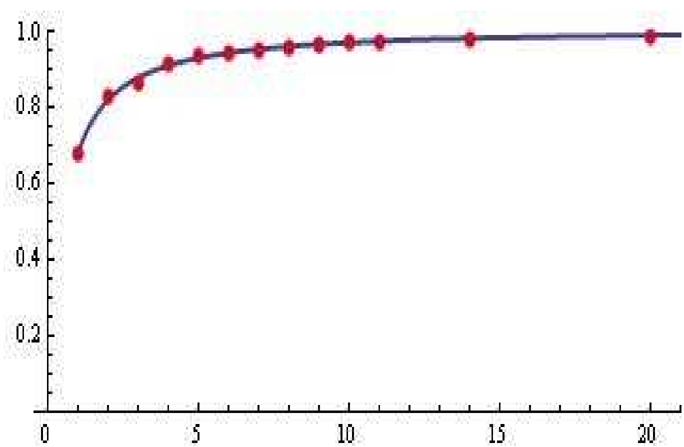

Figure 2: The fitted model (4) for $\alpha = 1.36443$, $\theta = 0.589538$, $\lambda = 0.1$ $\alpha_1 = 0.08$ $\beta_1 = 0.2999$ (Example 1).

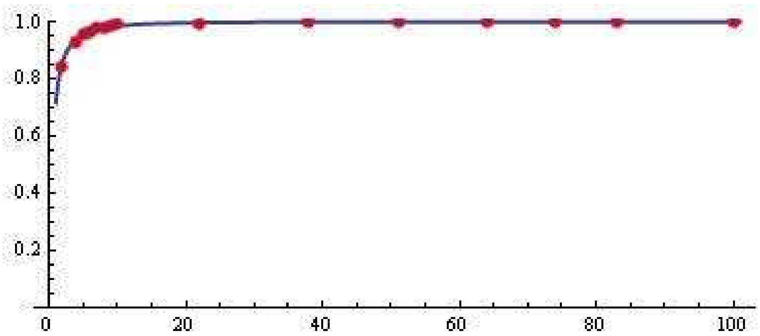

Figure 3: The fitted model (4) for $\alpha = 1.31203$, $\theta = 0.687361$, $\lambda = 0.1$ $\alpha_1 = 0.08$ $\beta_1 = 0.2993$ (Example 2).

2.2. APPROXIMATION OF SOME "SPECIFIC" DATA BY MODEL $M_2(T)$ - (5) WITH "POLYNOMIAL VARIABLE TRANSFER"

Example 3. Approximation of the data "MERS-CoV - 2012-2019" [6]

For the cumulative data the fitted model $M_2(t)$ for

$$n = 6,\ A = 2510,\ a_0 = 0,\ a_1 = 1.72163,\ a_2 = -32.2962,\ a_3 = 189.786$$

$$a_4 = -363.079,\ a_5 = 247.113,\ a_6 = -23.3546$$

is presented on Fig. 4.

Example 4. The following data is used in the modelling process [7] (see, Fig. 5)

For the actual values in the specified period the model $M_2(t)$ for

$$n = 6,\ A = 42,\ a_0 = 0,\ a_1 = -12.5293,\ a_2 = 40.8016,\ a_3 = -52.3756,$$

$$a_4 = 33.0529,\ a_5 = -10.1532,\ a_6 = 1.21949$$

is depicted on Fig. 6.

(We have adopted a scale on the horizontal axis: 0.1 division corresponds to 1 time interval).

2.3. THE MODEL $M_4(T)$ (7)

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the sigmoid - (7) satisfies the relation

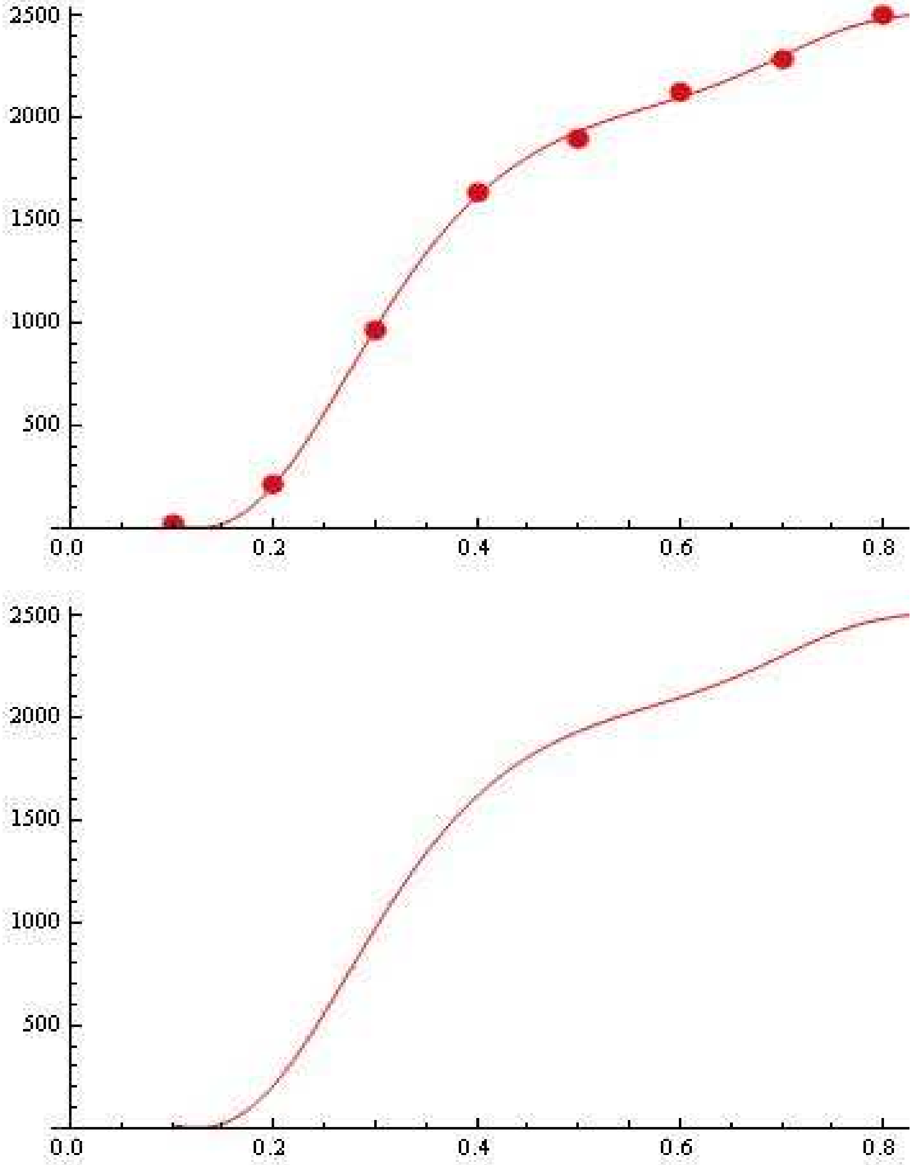

Figure 4: The fitted model $M_2(t)$ ($n = 6$, $A = 2510$, $a_0 = 0$, $a_1 = 1.72163$, $a_2 = -32.2962$, $a_3 = 189.786$, $a_4 = -363.079$, $a_5 = 247.113$, $a_6 = -23.3546$) for the data "MERS-CoV - 2012-2019" [6].

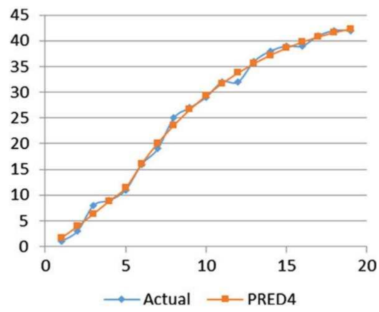

Figure 5: Goodness of fitcurve for release 4 [7].

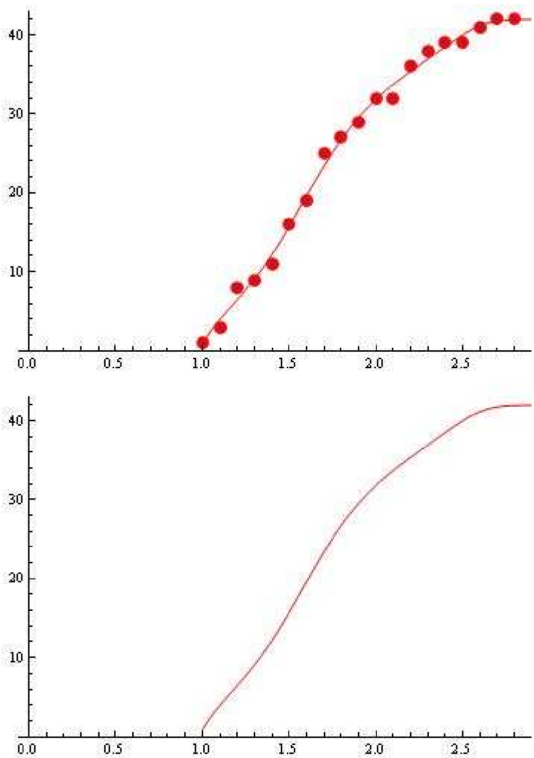

Figure 6: a) The "actual data"; b) The fitted model $M_2(t)$ ($n = 6$, $A = 42$, $a_0 = 0$, $a_1 = -12.5293$, $a_2 = 40.8016$, $a_3 = -52.3756$, $a_4 = 33.0529$, $a_5 = -10.1532$, $a_6 = 1.21949$) (Example 4).

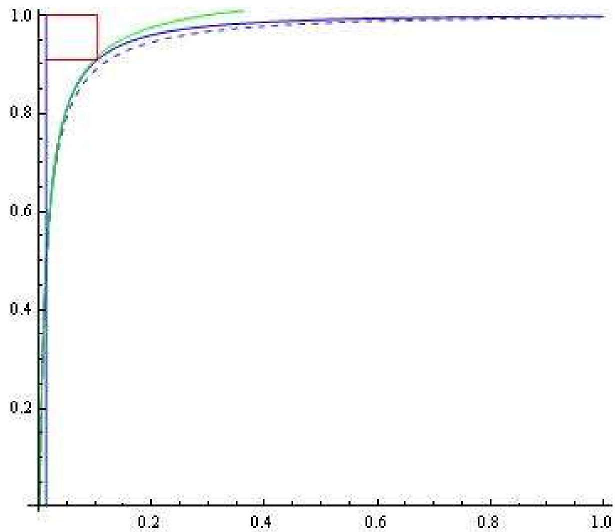

Figure 7: The model $M_4(t)$ (7) and two–sided estimations for $\alpha_1 = 0.08$, $\beta_1 = 0.2999$ and $\theta = 1.1$; $\lambda = 1.5$; $\alpha = 0.7$, $t_0 = 0.00137646$; H–distance $d = 0.0905713$.

$$M_4(t_0 + d) = 1 - d. \tag{10}$$

The model (7) and two–sided estimations for fixed $\alpha_1 = 0.08$, $\beta_1 = 0.2999$ and $\theta = 1.1$; $\lambda = 1.5$; $\alpha = 0.7$, $t_0 = 0.00137646$; H–distance $d = 0.0905713$ is plotted on Fig. 7.

2.4. APPROXIMATION OF THE DATA "MERS-COV - 2012-2019" [6] BY MODEL $M_5(T)$ - (8) WITH "POLYNOMIAL VARIABLE TRANSFER"

For the cumulative data (see, Example 3) the fitted model $M_5(t)$ for

$$n = 7,\ A = 2510,\ a_0 = 0,\ a_1 = 4.81202,\ a_2 = -91.9545,\ a_3 = 566.682$$

$$a_4 = -1322.19,\ a_5 = 1560.55,\ a_6 = -1081.51,\ a_7 = 417.983$$

is presented on Fig. 8.

In the cited article [8] He, Ahmad, Afify and Goual proposed some new models based on (ASE) approach, see Fig. 9. These models can be successfully expanded

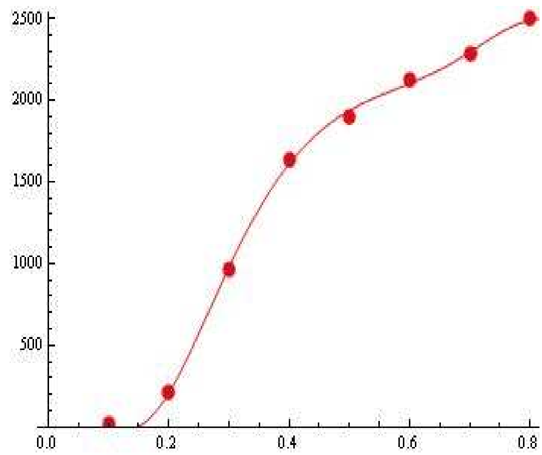

Figure 8: The fitted model $M_5(t)$ ($n = 7$, $A = 2510$, $a_0 = 0$, $a_1 = 4.81202$, $a_2 = -91.9545$, $a_3 = 566.682$, $a_4 = -1322.19$, $a_5 = 1560.55$, $a_6 = -1081.51$, $a_7 = 417.983$) for the data "MERS-CoV - 2012-2019" [6].

No.	Baseline model	Distribution function	Generated model
1	Weibull	$(2/\pi)\text{arcsine}(\{1-e^{-\gamma x^{\alpha}}\}^{a})$	ASE-Weibull
2	Lomax	$(2/\pi)\text{arcsine}(\{1-(1+\gamma x)^{-\alpha}\}^{a})$	ASE-Lomax
3	Uniform	$(2/\pi)\text{arcsine}(\{x/\theta\}^{a})$	ASE-Uniform
4	Linear failure rate	$(2/\pi)\text{arcsine}(\{1-e^{-\gamma x^{\alpha}-\theta x}\}^{a})$	ASE-Linear failure rate
5	Exponential	$(2/\pi)\text{arcsine}(\{1-e^{-\gamma x}\}^{a})$	ASE-Exponential
6	Rayleigh	$(2/\pi)\text{arcsine}(\{1-e^{-\gamma x^{2}}\}^{a})$	ASE-Rayleigh
7	Pareto	$(2/\pi)\text{arcsine}(\{1-(x_m/x)^{\alpha}\}^{a})$	ASE-Pareto
8	Burr	$(2/\pi)\text{arcsine}(\{1-(1+x^{k})^{-c}\}^{a})$	ASE-Burr
9	Topp Leone	$(2/\pi)\text{arcsine}(\{x^{\alpha}(2-x^{\alpha})\}^{a})$	ASE-Topp Leone
10	Log logistics	$(2/\pi)\text{arcsine}(\{1/1+(x/\gamma)^{-\alpha}\}^{a})$	ASE-Log logistics
11	Kumaraswamy	$(2/\pi)\text{arcsine}(\{1-(1-x^{a})^{b}\}^{\alpha})$	ASE-Kumaraswamy
12	Frechet	$(2/\pi)\text{arcsine}(\{e^{-(x/\gamma)^{-\alpha}}\}^{a})$	ASE-Frechet
13	Gamma	$(2/\pi)\text{arcsine}(\{(1/\Gamma\alpha)\gamma(\alpha,\beta x)\}^{a})$	ASE-Gamma
14	Lindely	$(2/\pi)\text{arcsine}(\{1-((e^{-\theta x}(1+\theta+\theta x))/(1+\theta))\}^{a})$	ASE-Lindely
15	Beta	$(2/\pi)\text{arcsine}(\{I_x(a,b)\}^{a})$	ASE-Beta
16	Normal	$(2/\pi)\text{arcsine}(\{\Phi(((x-\mu)/\sigma))\}^{a})$	ASE-Normal
17	Gumbel	$(2/\pi)\text{arcsine}(\{e^{-e^{-(x-\mu/\sigma)}}\}^{a})$	ASE-Gumbel
18	Power function	$(2/\pi)\text{arcsine}(\{(x/\gamma)^{\alpha}\}^{a})$	ASE-Power function
19	Half logistic	$(2/\pi)\text{arcsine}(\{(1-e^{-x})/(1+e^{-x})\}^{a})$	ASE-Half logistic
20	Erlang	$(2/\pi)\text{arcsine}(\{(1/(k-1)!)\gamma(k,\lambda x)\}^{a})$	ASE-Erlang
21	Lévy	$(2/\pi)\text{arcsine}(\{erfc(\sqrt{\sigma/2(x-\mu)})\}^{a})$	ASE-Lévy
22	Rice	$(2/\pi)\text{arcsine}(\{1-Q_1(v/\sigma,x/\sigma)\}^{a})$	ASE-Rice
23	Shifted Gompertz	$(2/\pi)\text{arcsine}(\{(1-e^{-bx})e^{-\eta e^{-bx}}\}^{a})$	ASE-shifted Gompertz
24	Dagum	$(2/\pi)\text{arcsine}(\{\{1+(x/\beta)^{-\alpha}\}^{-p}\}^{a})$	ASE-Dagum
25	Beta prime	$(2/\pi)\text{arcsine}(\{I_{(x/1+x)}(\alpha,\beta)\}^{a})$	ASE-Beta prime
26	Logistic	$(2/\pi)\text{arcsine}(\{1/1+e^{-(x-\mu/\sigma)}\}^{a})$	ASE-Logistic
27	Reciprocal	$(2/\pi)\text{arcsine}(\{(\log_e(x)-\log_e(\alpha))/(\log_e(\beta)-\log_e(\alpha))\}^{a})$	ASE-Reciprocal
28	Gompertz	$(2/\pi)\text{arcsine}(\{1-e^{-\eta(e^{bx}-1)}\}^{a})$	ASE-Gompertz
29	Hyperbolic secant	$(2/\pi)\text{arcsine}(\{(2/\pi)\arctan\{e^{(x\pi/2)}\}\}^{a})$	ASE-Hyperbolic secant

Figure 9: Some new models based on (ASE) approach, see [8].

in the light of our considerations - by inserting corrective corrections of the type "polynomial variable transfer" and here we will skip their analysis.

For other results, see [9]–[30].

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