Neural, Parallel, and Scientific Computations, **28**, No. 1 (2020), 27-35 ISSN: 1061-5369

ON THE "SATURATION" BY THE TYPE II TOPP–LEONE TRANSMUTED INVERTED KUMARASWAMY C.D.F.

ANNA MALINOVA[1], NIKOLAY PAVLOV[2],
TODORKA TERZIEVA[3], OLGA RAHNEVA[4]

[1,2,3]Faculty of Mathematics and Informatics
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA

[4]Faculty of Economy and Social Sciences
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA

ABSTRACT: In [1] the authors introduced the Type II Topp–Leone (transmuted) inverted Kumaraswamy (TIITLIK) distribution defined by combining the type II Topp–Leone–G family and the transmuted inverted Kumaraswamy distribution with c.d.f

$$F(t) = 1 - \left(1 - \left(1-(1+t)^{-a}\right)^{2b}\left(1+\lambda-\lambda\left(1-(1+t)^{-a}\right)^{b}\right)^{2}\right)^{\alpha}$$

where $\alpha, a, b \in R^+$ and $\lambda \in [-1, 1]$.

Also of interest to the specialists is the task of approximating the Heaviside function $h_{t_0}(t)$ where t_0 is the median with the new cumulative function in the Hausdorff sense [2].

The estimates of the value of the one–sided Hausdorff approximation obtained in this article can be used in practice as one possible additional criterion in "saturation" study.

We give example with real dataset: "data_*Witty*_*World*" [3] and "cancer data" [4]–[5].

Numerical examples are presented using *CAS MATHEMATICA*.

AMS Subject Classification: 41A46

Key Words: new Type II Topp–Leone (transmuted) inverted Kumaraswamy (TIITLIK) distribution, quantile function, "supersaturation" by the cumulative distribution function, Heaviside step–function, Hausdorff distance

Received: February 1, 2020 ; **Accepted:** April 17, 2020 ;
Published: May 28, 2020. **doi:** 10.12732/npsc.v28i1.3
Dynamic Publishers, Inc., Acad. Publishers, Ltd. https://acadsol.eu/npsc

1. INTRODUCTION

Definition 1. *In [1] the authors consider the Type II Topp–Leone (transmuted) inverted Kumaraswamy (TIITLIK) distribution defined by combining the type II Topp–Leone–G family and the transmuted inverted Kumaraswamy distribution.*

The corresponding cumulative distribution function is written as [1]:

$$M(t) = 1 - \left(1 - \left(1 - (1+t)^{-a}\right)^{2b} \left(1 + \lambda - \lambda \left(1 - (1+t)^{-a}\right)^{b}\right)^{2}\right)^{\alpha}, \qquad (1)$$

where $\alpha, a, b \in R^{+}$, $t > 0$ *and* $\lambda \in [-1, 1]$.

Definition 2. *The shifted Heaviside step function is defined by*

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases} \qquad (2)$$

Definition 3. *[2] The Hausdorff distance (the H–distance)* $\rho(f, g)$ *between two interval functions* f, g *on* $\Omega \subseteq \mathbb{R}$*, is the distance between their completed graphs* $F(f)$ *and* $F(g)$ *considered as closed subsets of* $\Omega \times \mathbb{R}$*. More precisely,*

$$\rho(f, g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\},$$

wherein $||.||$ *is any norm in* $\mathbb{R}^2$*, e. g. the maximum norm* $||(t, x)|| = \max\{|t|, |x|\}$*; hence the distance between the points* $A = (t_A, x_A)$*,* $B = (t_B, x_B)$ *in* $\mathbb{R}^2$ *is* $||A - B|| = max(|t_A - t_B|, |x_A - x_B|)$.

For other results, see [6]–[16].

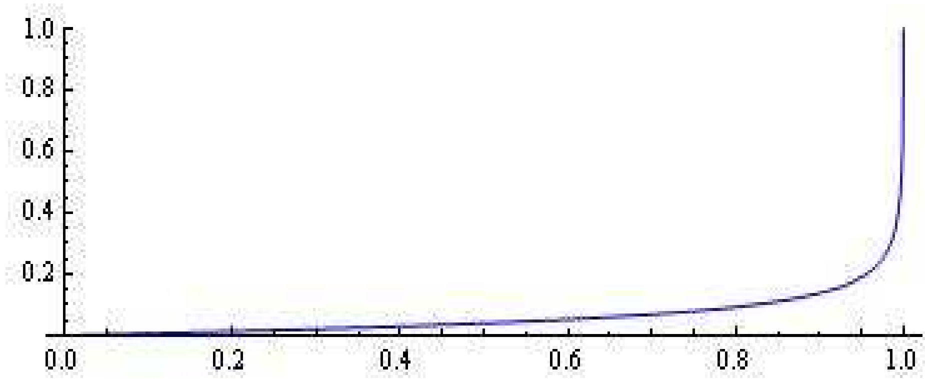

Figure 1: The function $Q(u)$ for $a = 13.1$, $b = 0.7$, $\alpha = 0.8$, $\lambda = 0.9$.

2. MAIN RESULTS

2.1. "SATURATION" BY THE MODEL (1)

In this Section we study the Hausdorff approximation [2] of the Heaviside step function $h_{t_0}(t)$ where t_0 is the "median" by families of the type (1).

The investigation of the characteristic "supersaturation" of the cdf (1) to the horizontal asymptote is important.

The quantile function is defined by [1]:

$$Q(u) = \left(\left(1 - \left(\frac{1+\lambda - \sqrt{1 + 2\left(1 - 2\sqrt{1-(1-u)^{\frac{1}{\alpha}}}\right)\lambda + \lambda^2}}{2\lambda} \right)^{\frac{1}{b}} \right)^{-\frac{1}{a}} - 1 \right) \tag{3}$$

(see, Fig. 1 for $a = 13.1$, $b = 0.7$, $\alpha = 0.8$, $\lambda = 0.9$).

The median is obtained by substituting $u = 0.5$ in (3).

Let t_0 is the value for which $F_1(t_0) = \frac{1}{2}$, i.e. $t_0 = Q(0.5)$.

The Hausdorff distance d between the function $h_{t_0}(t)$ and $M(t)$ satisfies the relation

$$M(t_0 + d) = 1 - d. \tag{4}$$

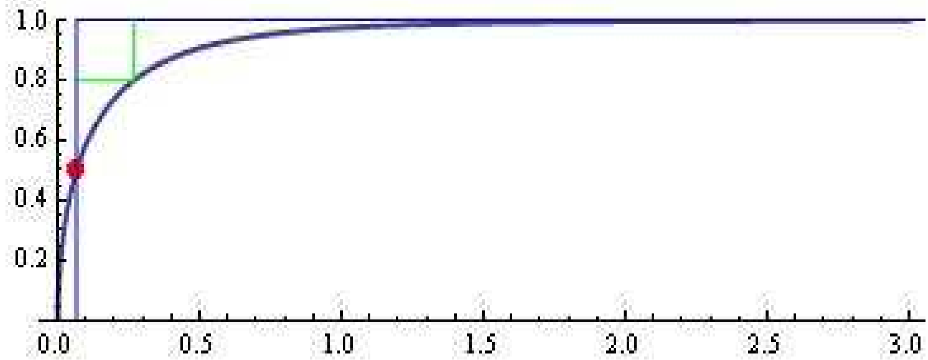

Figure 2: The model (1) for $a = 11.1$, $b = 0.3$, $\alpha = 0.4$, $\lambda = 0.6$, $t_0 = 0.0667413$; H–distance $d = 0.20121$.

For fixed $a = 11.1$, $b = 0.3$, $\alpha = 0.4$, $\lambda = 0.6$ we find $t_0 = 0.0667413$ and from the nonlinear equation (4) we have $d = 0.20121$ (see, Fig. 2).

For fixed $a = 13.1$, $b = 0.7$, $\alpha = 0.8$, $\lambda = 0.9$ we find $t_0 = 0.041368$ and from the nonlinear equation (4) we have $d = 0.0986378$ (see, Fig. 3).

From the graphics it can be seen that the "saturation" is faster.

Obviously, the investigations on the "supersaturation" gives the opportunity to the researcher for choice of appropriate model when approximating cumulative specific data in areas of Biostatistics, Population dynamics, Growth theory, Debugging and Test theory, Computer viruses propagation, Financial and Insurance mathematics.

2.2. APPLICATIONS

Example 1. Analysis of Witty worm infection behavior [3].

Here we will give an application of the model $M(t)$ when provide analysis of this real "data" [3].

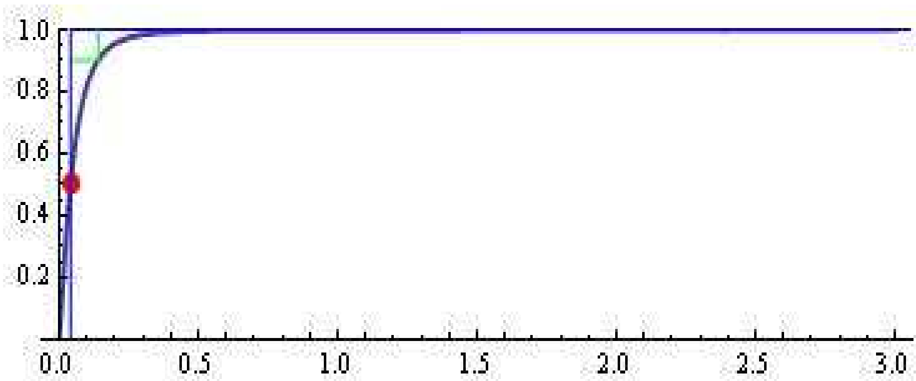

Figure 3: The model (1) for $a = 13.1$, $b = 0.7$, $\alpha = 0.8$, $\lambda = 0.9$, $t_0 = 0.041368$; H–distance $d = 0.0986378$.

$data\_Witty\_World =$

$\{\{0.1, 150\}, \{5, 869\}, \{10, 2141\}, \{15, 3637\}, \{20, 5312\},$

$\{26, 6602\}, \{31, 7562\}, \{36, 8340\}, \{41, 8941\}, \{46, 9389\}, \{51, 9734\},$

$\{56, 10060\}, \{61, 10349\}, \{66, 10586\}, \{71, 10800\}, \{76, 11169\},$

$\{86, 11362\}, \{91, 11532\}, \{96, 11684\}, \{101, 11823\}, \{106, 11972\},$

$\{111, 12118\}, \{116, 12256\}, \{121, 12372\}\}$

For entire World spreading parameters are (see Fig. 4)

$$\alpha = 0.507458; a = 1.7; b = 58; \lambda = 0.5; \omega = 13400.$$

where $M^*(t) = \omega M(t)$.

Example 2. We will illustrate the advances of the new family for approximation and modelling of "cancer data" (for some details see, [4]–[5]).

days	4	7	10	12	14	17	19	21
$R(t)$	0.415	0.794	1.001	1.102	1.192	1.22	1.241	1.3

Table 1: The "cancer data" [4]–[5]

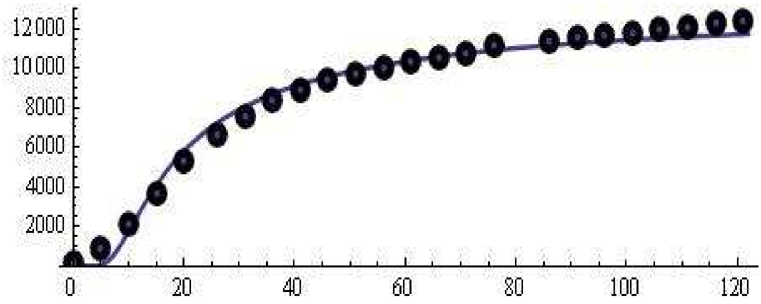

Figure 4: The model M^*.

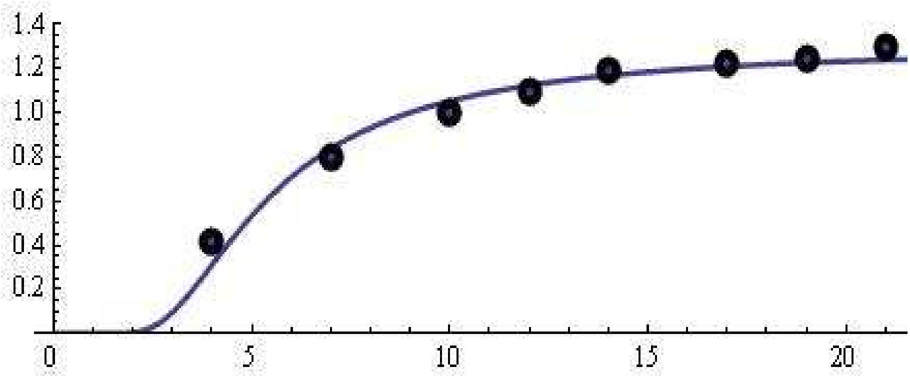

Figure 5: The model $M^*(t)$ based on the "cancer data".

The model $M^*(t) = \omega M(t)$ based on the data from Table 1 for the estimated parameters:

$$\omega = 1.3;\ a = 2.55;\ b = 50;\ \alpha = 0.815341;\ \lambda = 0.4$$

is plotted on Fig. 5.

In addition, the experiments must be performed with very high accuracy.

Obviously, the model $M(t)$ can also be used for approximating of some "specific data".

For some approximation, computational and modelling aspects, see [17]–[31].

Acknowledgment

This paper is supported by the National Scientific Program "Information and Communication Technologies for a Single Digital Market in Science, Education and Security (ICTinSES)", financed by the Ministry of Education and Science.

REFERENCES

[1] R. ZeinEldin, F. Jamal, Ch. Chesneau, M. Elgarhy, Type II Topp–Leone inverted Kumaraswamy distribution with statistical inference and applications, *Symmetry*, **11** (2019).

[2] B. Sendov, *Hausdorff Approximations*, Kluwer, Boston (1990).

[3] C. Shannon, D. Moore, The Spread of the Witty Worm, *IEEE Security & Privacy*, **July/August**, (2004), 46–50.

[4] M. Vinci, S. Gowan, F. Boxall, L. Patterson, M. Zimmermann, W. Court, C. Lomas, M. Mendila, D. Hardisson, S. Eccles, Advances in establishment and analysis of three-dimensional tumor spheroid-based functional assays for target validation and drug evaluation, *BMC Biology*, **10** (2012).

[5] A. Antonov, S. Nenov, T. Tsvetkov, Impulsive controllability of tumor growth, *Dynamic Systems and Appl.*, **28**, No. 1 (2019), 93–109.

[6] G. W. Topp, F. C. Leone, A family of J–shaped frequency functions, *Journal of American Stat. Association*, **50** (1955), 209–219.

[7] Rashad A.R. Bantan, Farrukh Jamal, Christophe Chesneau, Mohammed Elgarhy, A New Power Topp-Leone Generated Family of Distributions with Applications, *Entropy* (2019).

[8] M. Elgarhy, M. Nazir, F. Jamal, G. Ozel, The type II Topp–Leone generated family of distributions: properties and applications, *J. of Statistics and Management Systems*, **21**, No. 8 (2018), 1525–155.

[9] M. Tahir, G. Cordeiro, Compounding of distributions: A survey and new generalized classes, *J. of Stat. Distr. and Appl.,* **3**, No. 13 (2016), 1–35.

[10] A. Hassan, M. Elgarhy, Z. Ahmad, Type II generalized Topp–Leone family of distributions: properties and applications, *J. of Data Sci.,* **17**, No 4 (2019), 638–659.

[11] A. Mahdavi, Generalized Topp–Leone family of distributions, *J. of Biost. and Epidemiology,* **3**, No 2 (2017), 65–75.

[12] H. Reyad, M. Alizadeh, F. Jamal, S. Othman, G. Hamedani, The exponentiated generalized Topp Leone–G family of distributions: properties and applications, *Pak. J. of Stat. and Oper. Res.,* **15**, No 1 (2019), 1–24.

[13] H. Reyad, M. Korkmaz, A. Afify, G. Hamedani, S. Othman, The Frechet Topp Leone–G family of distributions: properties, characterizations and applications, *Ann. of Data Sci.,* (2019).

[14] S. Rezaei, B. Sadr, M. Alizadeh, S. Nadarajah, Topp–Leone generated family of distributions: properties and applications, *Comm. in Stat.–Theory and Methods,* **46**, No 6 (2016), 2893–2909.

[15] H. Yousof, M. Alizadeh, S. Jahanshalu, T. Ramires, I. Ghosh, G. Hamedani, The transmuted Topp–Leone G family of distributions: theory, characterizations and applications, *J. of Data Sci.,* **15** (2017), 723–740.

[16] A. Iliev, A. Rahnev, N. Kyurkchiev, S. Markov, A Study on the Unit-logistic, Unit-Weibull and Topp-Leone Cumulative Sigmoids, *Biomath Communications*, **6**, No 1 (2019), 1–15.

[17] N. Kyurkchiev, S. Markov, On the Hausdorff distance between the Heaviside step function and Verhulst logistic function, *J. Math. Chem.*, **54** No. 1 (2016), 109–119.

[18] N. Kyurkchiev, S. Markov, *Sigmoid functions: Some Approximation and Modelling Aspects*, LAP LAMBERT Academic Publishing, Saarbrucken (2015), ISBN: 978-3-659-76045-7.

[19] N. Kyurkchiev, A. Iliev, S. Markov, *Some Techniques for Recurrence Generating of Activation Functions: Some Modeling and Approximation Aspects*, LAP LAMBERT Academic Publishing (2017), ISBN: 978-3-330-33143-3.

[20] S. Markov, A. Iliev, A. Rahnev, N. Kyurkchiev, A note on the Log-logistic and transmuted Log-logistic models. Some applications, *Dynamic Systems and Applications*, **27** No. 3 (2018), 593–607.

[21] S. Markov, N. Kyurkchiev, A. Iliev, A. Rahnev, On the approximation of the cut functions by hyper-log-logistic function, *Neural, Parallel and Scientific Computations*, **26** No. 2 (2018), 169–182.

[22] N. Kyurkchiev, A. Iliev, *Extension of Gompertz-type equation in modern science. 240 Anniversary of the birth of B. Gompertz*, LAP LAMBERT Academic Publishing, (2018), ISBN: 978-613-9-90569-0.

[23] N. Kyurkchiev, A. Iliev, A. Rahnev, *Some Families of Sigmoid Functions: Applications to Growth Theory*, LAP LAMBERT Academic Publishing, (2019), ISBN: 978-613-9-45608-6.

[24] A. Iliev, N. Kyurkchiev, S. Markov, On the Approximation of the step function by some sigmoid functions, *Mathematics and Computers in Simulation*, **133** (2017), 223–234.

[25] S. Markov, N. Kyurkchiev, A. Iliev, A. Rahnev, On the approximation of the generalized cut functions of degree $p+1$ by smooth hyper-log-logistic function, *Dynamic Systems and Applications*, **27** No. 4 (2018), 715–728.

[26] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Some software reliability models: Approximation and modeling aspects*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-82805-0.

[27] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Nontrivial Models in Debugging Theory (Part 2)*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-87794-2.

[28] A. Iliev, N. Kyurkchiev, A. Rahnev, T. Terzieva, *Some models in the theory of computer viruses propagation*, LAP LAMBERT Academic Publishing (2019), ISBN: 978-620-0-00826-8.

[29] A. Malinova, A. Golev, O. Rahneva, V. Kyurkchiev, Some Notes on the Kumaraswamy-Weibull-Exponential Cumulative Sigmoid, *International Journal of Pure and Applied Mathematics*, **120**, No. 4 (2018), 521–529.

[30] N. Kyurkchiev, Uniform Approximation of the Generalized Cut Function by Erlang Cumulative Distribution Function. Application in Applied Insurance Mathematics, *International Journal of Theoretical and Applied Mathematics*, **2**, No. 2 (2016), 40-44.

[31] N. Kyurkchiev, Mathematical Concepts in Insurance and Reinsurance: Some Moduli in Programming Environment MATHEMATICA, LAP LAMBERT Academic Publishing, 2016, ISBN:978-3-659-96906-5.