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# INVESTIGATIONS ON A NEW SOFTWARE RELIABILITY MODEL. II

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**ABSTRACT:** In [1] Aggarwal, Gandhi, Verma and Tandon considered a new four–parameter expected mean number of faults - function $M(t)$ by:

$$M(t) = \frac{a}{1-\alpha}\left(1 - (1+bt)^{r(1-\alpha)}e^{-btr(1-\alpha)}\right),$$

where $\alpha$ is the constant rate at which new faults are introduced.

Also of interest to the specialists is the task of approximating the Heaviside function

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases}$$

where $t_0$ is the median, i.e. $M(t_0) = \frac{1}{2}$ with the new function in the Hausdorff sense.

We give example with real dataset.

Numerical examples, illustrating our results are presented using programming environment *CAS Mathematica*.

**AMS Subject Classification:** 41A46

**Key Words:** four–parameter expected mean number of faults, four–parameter Aggarwal, Gandhi, Verma and Tandon's software reliability model, Heaviside step–function $h_{t_0}(t)$, Hausdorff distance

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## 1. INTRODUCTION AND PRELIMINARIES

Reliability modelling is a process of determining an appropriate mathematical expression which can describe the time–based software failure process.

Some software reliability models and studies on their "intrinsic properties", can be found in [3]–[43], [57].

In this note we study the Hausdorff approximation of the Heaviside function $h_{t_0}(t)$ by function $M(t)$, defined by Aggarwal, Gandhi, Verma and Tandon.

The model have been tested with real-world data.

**Definition 1.** *Aggarwal, Gandhi, Verma and Tandon [1] developed the following new function:*

$$M(t) = \frac{a}{1-\alpha}\left(1 - (1+bt)^{r(1-\alpha)} e^{-btr(1-\alpha)}\right), \tag{1}$$

*where $\alpha$ is the constant rate at which new faults are introduced.*

**Definition 2.** *The shifted Heaviside step function is defined by*

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases} \tag{2}$$

**Definition 3.** *[2] The Hausdorff distance (the H–distance) $\rho(f,g)$ between two interval functions $f, g$ on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,*

$$\rho(f,g) = \max\{ \sup_{A\in F(f)} \inf_{B\in F(g)} ||A-B||, \sup_{B\in F(g)} \inf_{A\in F(f)} ||A-B||\},$$

*wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A-B|| = max(|t_A - t_B|, |x_A - x_B|)$.*

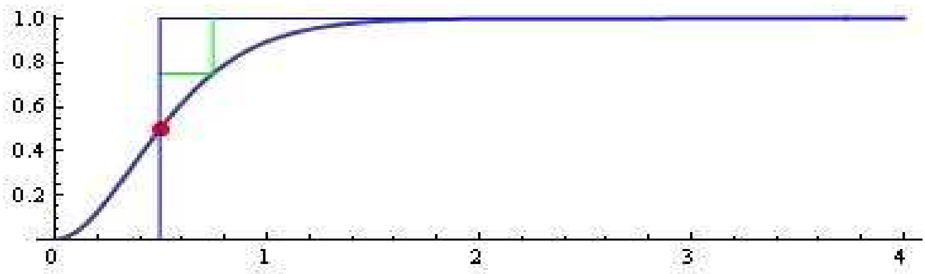


Figure 1: The model (1) for $a = 0.9$, $b = 1.1$, $\alpha = 0.1$, $r = 7$ and $t_0 = 0.495576$; H–distance $d = 0.24896$.

## 2. MAIN RESULTS

### 2.1. A NOTE ON THE NEW AGGARWAL, GANDHI, VERMA AND TANDON'S SOFTWARE RELIABILITY GROWTH MODEL (1)

The investigation of the characteristic ”supersaturation” of the model (1) to the horizontal asymptote is important.

Let $t_0$ is the value for which $M(t_0) = \frac{1}{2}$.

The one–sided Hausdorff distance $d$ between the function $h_{t_0}(t)$ and the $M(t)$ satisfies the relation

$$M(t_0 + d) = 1 - d. \tag{3}$$

For given $\alpha$, $a$, $b$, $r$ and $t_0$, the nonlinear equation $M(t_0+d)-1+d = 0$ has unique positive root – $d$.

The model (1) for $a = 0.9$, $b = 1.1$, $\alpha = 0.1$, $r = 7$ and $t_0 = 0.495576$ is visualized on Fig. 1.

From the nonlinear equation (3) we have: $d = 0.24896$.

The model (1) for $a = 0.99$, $b = 1.7$, $\alpha = 0.01$, $r = 10$ and $t_0 = 0.248392$ is visualized on Fig. 2.

From the nonlinear equation (3) we have: $d = 0.173821$.

The model (1) for $a = 0.999$, $b = 2$, $\alpha = 0.001$, $r = 20$ and $t_0 = 0.120403$ is visualized on Fig. 3.

From the nonlinear equation (3) we have: $d = 0.136971$.

Some computational examples are presented in Table 1: ”The saturation with the model (1) in Hausdorff sence”.

From the above examples, it can be seen that the ”supersaturation” by the $M(t)$ is faster.

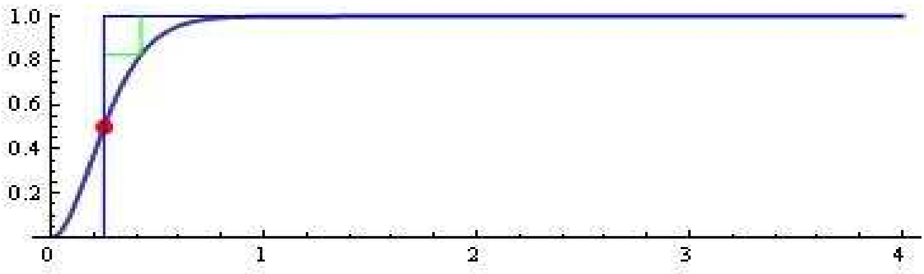


Figure 2: The model (1) for $a = 0.99$, $b = 1.7$, $\alpha = 0.01$, $r = 10$ and $t_0 = 0.248392$; H–distance $d = 0.173821$.

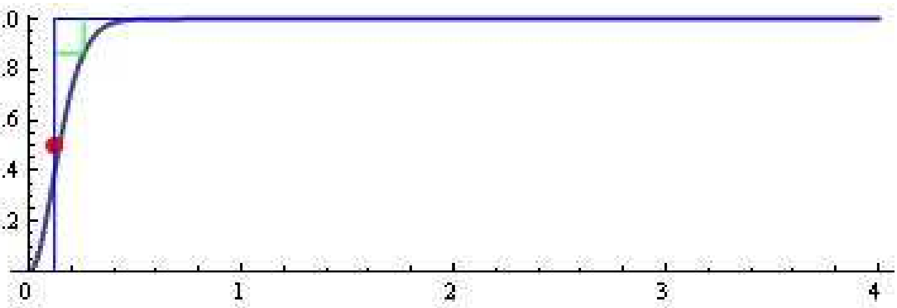


Figure 3: The model (1) for $a = 0.999$, $b = 2$, $\alpha = 0.001$, $r = 20$ and $t_0 = 0.120403$; H–distance $d = 0.136971$.

Obviously, this "advantage" can actually be used to approximate some specific data.

In the next Section, we will support what is said by analyzing real dataset: "actual data to estimate the number of software residual faults" [44]–[45].

## 2.2. APPLICATION

We analyze the following "actual data to estimate the number of software residual faults" [44]–[45] (see, Fig. 4).

| $a$ | $b$ | $\alpha$ | $r$ | $t_0$ | $H - distance$ |
|---|---|---|---|---|---|
| 0.9 | 1.1 | 0.1 | 7 | 0.495576 | 0.24896 |
| 0.99 | 1.7 | 0.01 | 10 | 0.248392 | 0.173821 |
| 0.999 | 2 | 0.001 | 20 | 0.120403 | 0.136971 |
| 0.999 | 2.5 | 0.001 | 25 | 0.101782 | 0.0969141 |
| 0.9999 | 2.6 | 0.0001 | 30 | 0.0887109 | 0.0876929 |
| 0.9999 | 3 | 0.0001 | 40 | 0.0659675 | 0.07056 |
| 0.9999 | 4 | 0.0001 | 50 | 0.0439719 | 0.0520972 |

Table 1: "The saturation with the model (1) in Hausdorff sense". The Hausdorff distance $d$ computed by nonlinear equation (3)

$data\_Satoh :=$

$\{\{1, 248\}, \{2, 262\}, \{3, 372\}, \{4, 526\}, \{5, 742\},$
$\{6, 958\}, \{7, 1215\}, \{8, 1471\}, \{9, 1738\}, \{10, 1936\},$
$\{11, 1971\}, \{12, 2147\}, \{13, 2258\}, \{14, 2418\}, \{15, 2567\},$
$\{16, 2688\}, \{17, 2809\}, \{18, 2925\}, \{19, 3026\}, \{20, 3205\},$
$\{21, 3348\}, \{22, 3476\}, \{23, 3573\}, \{24, 3719\}, \{25, 3750\},$
$\{26, 3952\}, \{27, 4048\}, \{28, 4137\}, \{29, 4251\}, \{30, 4301\},$
$\{31, 4351\}, \{32, 4401\}, \{33, 4439\}, \{34, 4488\}, \{35, 4548\},$
$\{36, 4596\}, \{37, 4629\}, \{38, 4680\}, \{39, 4713\}, \{40, 4749\},$
$\{41, 4783\}, \{42, 4817\}, \{43, 4849\}, \{44, 4877\}, \{45, 4901\},$
$\{46, 4928\}, \{47, 4950\}, \{48, 4970\}, \{49, 4998\}, \{50, 5024\},$
$\{51, 5060\}, \{52, 5085\}, \{53, 5088\}, \{54, 5090\}, \{55, 5110\},$
$\{56, 5129\}, \{57, 5139\}, \{58, 5167\}, \{59, 5186\}\}.$

After that using the model $M(t)$ for $\alpha = 0.1$, $a = 4667.4$, $b = 0.286409$ and $r = 0.299832$ we obtain the fitted model (see, Fig. 4).

**Remark.** In many cases it is appropriate to use the following model by Diwakar, A. G. Aggarwal [42]:

$$M_1(t) = \frac{a}{1-\alpha}\left(1 - e^{-b(1-\alpha)\frac{t^{k+1}}{k+1}}\right), \tag{4}$$

where $\alpha$ is the constant rate at which new faults are introduced.

After that using the model $M_1(t)$ for $\alpha = 0.1$, $a = 4667.4$, $b = 0.0293167$ and $k = 0.306508$ we obtain the fitted model (see, Fig. 5).

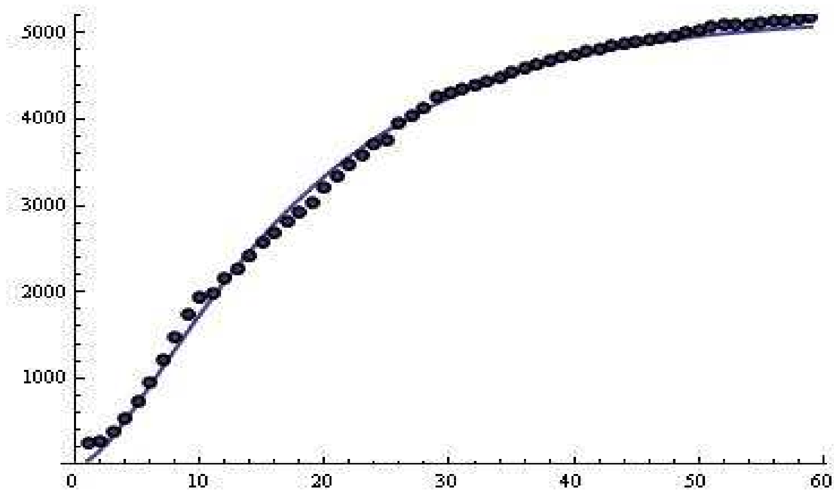


Figure 4: The fitted model $M(t)$.

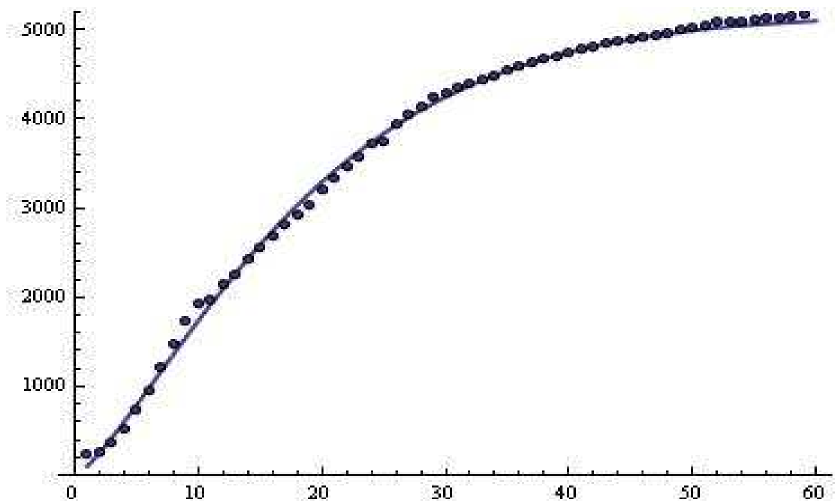


Figure 5: The fitted model $M_1(t)$.

Fig. 4 and Fig. 5 shows that the $M(t)$ and $M_1(t)$ models used are comparable, with a slight advantage in approximating the specific database in favor of the $M(t)$ model.

## 3. CONCLUDING REMARKS

The analysis we conducted in this article on the new Diwakar and Aggarwal's model shows its advantages and reliability compared to other similar models.

For other approximation and modelling results, see [46]–[56].

We hope that the results will be useful for specialists in this scientific area.

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