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INVESTIGATIONS ON A "EXPONENTIAL–EXPONENTIATED–EXPONENTIAL" GROWTH MODEL

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ABSTRACT: In this paper we study the characteristic - "saturation" of the cumulative distribution function of the "Type I General Class of Distributions" proposed by *Hamediani et al.* [1] to the horizontal asymptote in the Hausdorff sense.

We also analyze some experimental data.

The experiments show that in some cases the use of the model proposed in [1] and analyzed in this article with "respect to the Hausdorff distance" is satisfactory.

Numerical examples, illustrating our results are presented using programming environment *CAS Mathematica*.

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Key Words: Type I general exponential–exponentiated–exponential model (EEEM), Heaviside function, Hausdorff approximation

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1. INTRODUCTION AND PRELIMINARIES

Definition 1. The cdf of the "Type I general exponential–exponentiated–exponential distribution" is defined by [1]

$$M(t) = e^{\lambda(1-((1-e^{-\frac{t}{\theta}})^{b})^{-\alpha})}, \tag{1}$$

for $t \geq 0$, and $\lambda > 0$, $b > 0$, $\theta > 0$, $\alpha > 0$.

Some properties and applications can be found in [1]–[3].

Definition 2. The *shifted Heaviside step function* is defined by

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases}$$

Definition 3. The Hausdorff distance [4] (the H–distance) $\rho(f,g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$.

More precisely,

$$\rho(f,g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|, |x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

We study the Hausdorff approximation of the *shifted Heaviside step function* by the family of type (1).

2. MAIN RESULTS AND NUMERICAL EXAMPLES

We consider the class of this family:

$$M(t_0) = \frac{1}{2}; \quad t_0 = -\theta \ln \left(1 - \left(1 + \frac{\ln 2}{\lambda}\right)^{-\frac{1}{b\alpha}}\right). \tag{2}$$

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the sigmoid (1)–(2) satisfies the relation

$$M(t_0 + d) = 1 - d. \tag{3}$$

For given $\lambda > 0$, $b > 0$, $\theta > 0$, $\alpha > 0$ the nonlinear equation $M(t_0 + d) - 1 + d = 0$ has unique positive root – d.

The model (1)–(2) for $\lambda = 1.5$, $b = 0.5$, $\theta = 0.3$, $\alpha = 0.4$ and $t_0 = 0.0486371$ is visualized on Fig. 1.

From the nonlinear equation (3) we have: $d = 0.181134$.

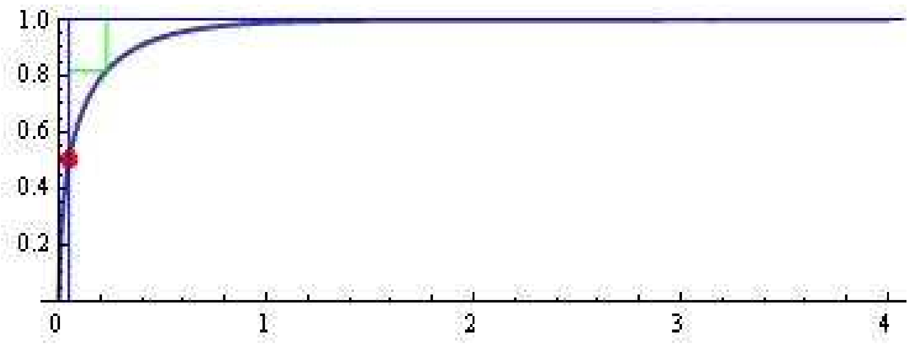

Figure 1: The model (1)–(2) for $\lambda = 1.5$, $b = 0.5$, $\theta = 0.3$, $\alpha = 0.4$ and $t_0 = 0.0486371$; H–distance $d = 0.181134$.

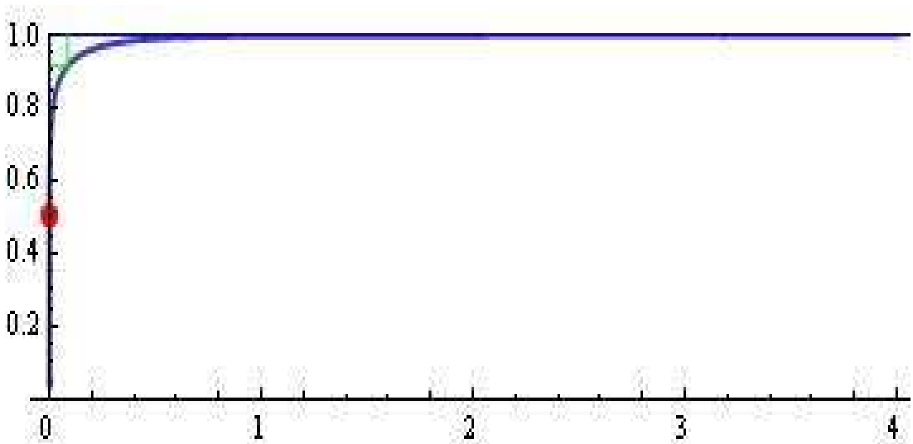

Figure 2: The model (1)–(2) for $\lambda = 0.5$, $b = 0.9$, $\theta = 0.25$, $\alpha = 0.15$ and $t_0 = 0.0003984$; H–distance $d = 0.0862234$.

The model (1)–(2) for $\lambda = 0.5$, $b = 0.9$, $\theta = 0.25$, $\alpha = 0.15$ and $t_0 = 0.0003984$ is visualized on Fig. 2.

From the nonlinear equation (3) we have: $d = 0.0862234$.

Some computational examples are presented in Table 1.

From the above examples, it can be seen that the "super saturation" is faster.

1. We examine the following data for the growth of red abalone Haliotis Rufescens in Northern California (see, Fig. 3 [5])

The fitted model $M^{ast}(t) = \omega M(t)$ for $\lambda = 0.754$, $b = 0.775696$, $\theta = 18.6101$, $\alpha = 0.749892$ and $\omega = 270$ is visualized on Fig. 4.

2. Analysis of data "growth of the cumulative number of TREZ publications" [6],

α	b	λ	θ	t_0	$H - distance$
0.4	0.5	1.5	0.3	0.0486371	0.181134
0.4	0.5	0.9	0.25	0.0148142	0.13983
0.5	0.6	1	0.1	0.0189776	0.103801
0.15	0.9	0.5	0.25	0.0003984	0.0862234
0.2	0.7	0.4	0.15	0.00011417	0.0627285
0.12	0.6	0.7	0.15	0.000010586	0.0579859

Table 1: The Hausdorff distance d computed by nonlinear equation (3)

Age	$Length(mm)$
1	16.1
2	33.9
3	54.3
4	76.2
5	97.8
6	117.1
7	133.3
8	146.5
9	157.2
10	166
11	173.3
12	179.6
13	185
14	189.9
15	194

Figure 3: The extended data for modeling the growth of red abalone Haliotis Rufescens in Northern California.

[7]

$$data\_Journal$$

$$:= \{\{1.1, 5\}, \{2, 37\}, \{3, 107\}, \{4, 201\}, \{5, 298\}, \{6, 439\},$$

$$\{7, 617\}, \{8, 773\}, \{9, 936\}, \{10, 1121\}, \{11, 1316\},$$

$$\{12, 1451\}, \{13, 1563\}, \{14, 1629\}, \{15, 1722\}, \{16, 1788\}\};$$

After that using the model $M^*(t) = \omega M(t)$ for $\lambda = 4.24$, $b = 0.957212$, $\theta = 4.77001$, $\alpha = 0.954099$ and $\omega = 2000$ we obtain the fitted model (see, Fig. 5).

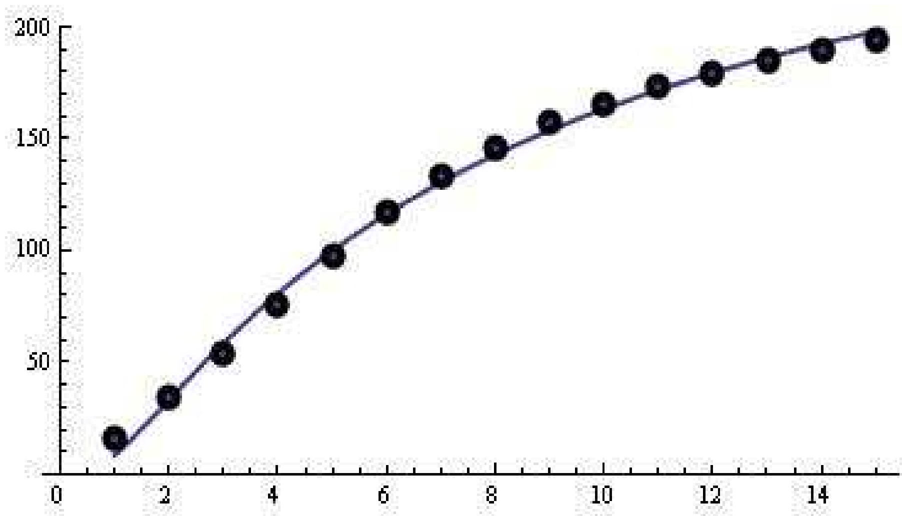

Figure 4: The fitted model $M^*(t)$.

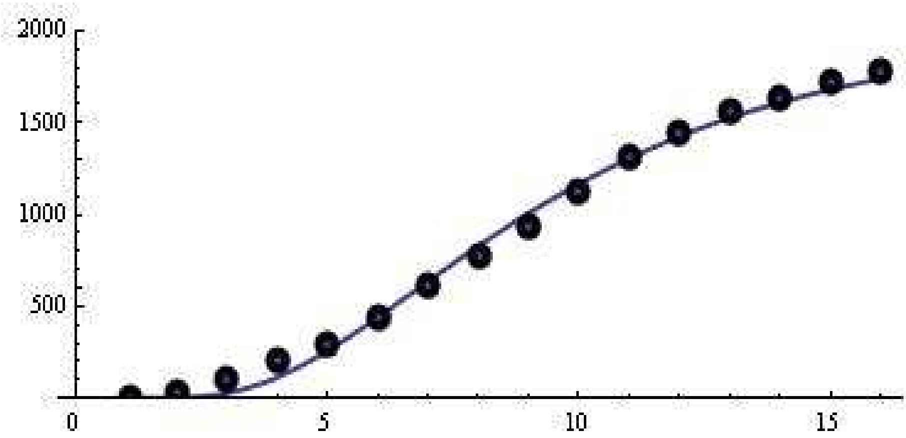

Figure 5: The fitted models $M^*(t)$.

3. CONCLUSIONS

The proposed growth model can be successfully used with success (of course, after extensive research) in the field of analysis of Computer Viruses Propagation, Biochemical sciences and Debugging and Test Theory.

For some approximation, computational and modelling aspects, see [8]–[29].

The experiments show that in some cases the use of the model proposed in [1] and analyzed in this article with "respect to the Hausdorff distance" is satisfactory.

Specialists working in this scientific field have a say.

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