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# THE LOMAX–D–GENERALIZED–WEIBULL CUMULATIVE SIGMOID WITH APPLICATIONS TO THE THEORY OF COMPUTER VIRUSES PROPAGATION. IV

TODORKA TERZIEVA[1], ANTON ILIEV[2],
ASEN RAHNEV[3], AND NIKOLAY KYURKCHIEV[4]

[1,2,3,4]Faculty of Mathematics and Informatics
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA

**ABSTRACT:** In [1] the authors look at a new probability model, which is a remarkable combination of the Lomax and generalized Weibull distributions based on an exponent odd function.

The authors' assertion that probability distribution produces very good results in approximating specific data from the field of hydrology has encouraged us to conduct further studies on "saturation" in Hausdorff sense of the corresponding commutative function to the horizontal asymptote hoping to partially contribute to uncovering some of the "intrinsic properties" of this apparently good model.

We will show that the proposed model can be successfully used with success (of course, after extensive research) in the field of analysis of Computer Viruses Propagation.

We also analyze some experimental data: the cumulative number of Welchia attackers; data of Conficker propagation in 2008; the cumulative number of users attacked by Trojan-Ransom malware.

Numerical examples, illustrating our results are presented using programming environment *CAS Mathematica*.

**AMS Subject Classification:** 41A46

**Key Words:** Lomax–D–generalized Weibull cumulative sigmoid, Heaviside step–function $h_{t_0}(t)$, Hausdorff distance

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## 1. INTRODUCTION AND PRELIMINARIES

In this note we study the Hausdorff approximation of the Heaviside function $h_{t_0}(t)$ by the cdf of the Lomax–D–generalized Weibull distribution (cdfLDGW), defined by Hussain, Bakouch and Chesneau [1].

The model have been tested with real-world data.

**Definition 1.** *Hussain, Bakouch and Chesneau [1] developed the following cdf of the new Lomax–D–generalized Weibull distribution for* $t \geq 0$*:*

$$M(t) = 1 - \left(1 + \frac{e^{\theta((1-\lambda t^{\theta})^{-\frac{1}{\lambda}}-1)} - 1}{e^{\theta} - 1}\right)^{-\beta} \tag{1}$$

*where* $\lambda \leq 0$, $\theta > 0$, $\beta > 0$.

**Definition 2.** *The shifted Heaviside step function is defined by*

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases} \tag{2}$$

**Definition 3.** *[2] The Hausdorff distance (the H–distance)* $\rho(f, g)$ *between two interval functions* $f, g$ *on* $\Omega \subseteq \mathbb{R}$*, is the distance between their completed graphs* $F(f)$ *and* $F(g)$ *considered as closed subsets of* $\Omega \times \mathbb{R}$*. More precisely,*

$$\rho(f, g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\},$$

*wherein* $||.||$ *is any norm in* $\mathbb{R}^2$*, e. g. the maximum norm* $||(t, x)|| = \max\{|t|, |x|\}$*; hence the distance between the points* $A = (t_A, x_A)$*,* $B = (t_B, x_B)$ *in* $\mathbb{R}^2$ *is* $||A - B|| = max(|t_A - t_B|, |x_A - x_B|)$.

## 2. MAIN RESULTS

### 2.1. A NOTE ON THE NEW (CDFLDGW)

The investigation of the characteristic ”supersaturation” of the cdf (1) to the horizontal asymptote is important.

Let $t_0$ is the value for which $M(t_0) = \frac{1}{2}$.

The one–sided Hausdorff distance $d$ between the function $h_{t_0}(t)$ and the cdf (1) satisfies the relation

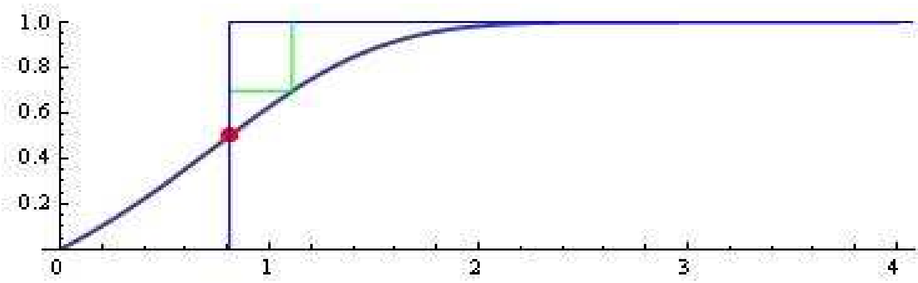


Figure 1: The cdf (1) for $\lambda = -0.05$, $\beta = 0.8$, $\theta = 0.99$ and $t_0 = 0.809797$; H–distance $d = 0.301775$.

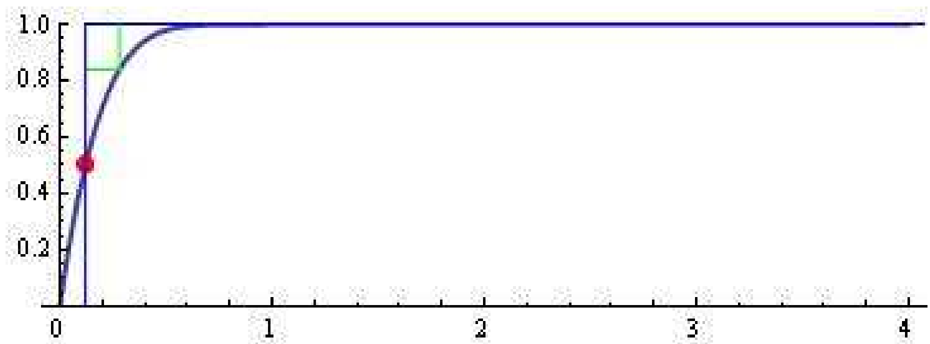


Figure 2: The cdf (1) for $\lambda = -0.001$, $\beta = 8.9$, $\theta = 0.99$ and $t_0 = 0.119372$; H–distance $d = 0.161348$.

$$M(t_0 + d) = 1 - d. \tag{3}$$

For given $\lambda$, $\beta$, $\theta$ and $t_0$, the nonlinear equation $M(t_0 + d) - 1 + d = 0$ has unique positive root $-$ $d$.

The cdf (1) for $\lambda = -0.05$, $\beta = 0.8$, $\theta = 0.99$ and $t_0 = 0.809797$ is visualized on Fig. 1.

From the nonlinear equation (3) we have: $d = 0.301775$.

The cdf (1) for $\lambda = -0.001$, $\beta = 8.9$, $\theta = 0.99$ and $t_0 = 0.119372$ is visualized on Fig. 2.

From the nonlinear equation (3) we have: $d = 0.161348$.

The cdf (1) for $\lambda = -0.0005$, $\beta = 25$, $\theta = 0.999$ and $t_0 = 0.0459322$ is visualized on Fig. 3.

From the nonlinear equation (3) we have: $d = 0.097724$.

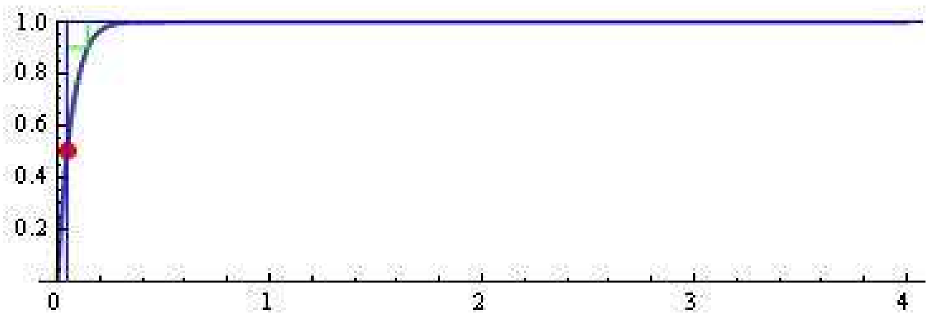


Figure 3: he cdf (1) for $\lambda = -0.0005$, $\beta = 25$, $\theta = 0.999$ and $t_0 = 0.0459322$; H–distance $d = 0.097724$.

From the above examples, it can be seen that the "supersaturation" by the (cdf) $M(t)$ is faster.

Obviously, this "advantage" can actually be used to approximate some specific data from the field of analysis of Computer Viruses Propagation.

In the next Section, we will support what is said by analyzing real datasets.

## 2.2. APPLICATIONS

Welchia worm and Cryptolocker ransomware have a long growing phase in contrast to many other threats.

In September 2013 the CryptoLocker malware starting its invasion using mainly P2P ZeuS (aka Gameover ZeuS) malware. CryptoLocker' main aim was to receive money from the unsuspecting victims for decrypting their files.

Welchia worm uses a vulnerability in the Microsoft remote procedure call service. Welchia firstly checks for Blaster worm and if it is exists continues with Blaster deletion as well as takes care for computer to be immunised for Blaster worm.

**Example 1.** Analysis of Welchia worm infection behavior

For epidemic as Welchia worm it is appropriately to use a model

$$M^*(t) = \omega \left( 1 - \left( 1 + \frac{e^{\theta((1-\lambda t^\theta)^{-\frac{1}{\lambda}}-1)} - 1}{e^\theta - 1} \right)^{-\beta} \right) \tag{4}$$

for approximating data from the statistics collected on an individual Welchia [3] honeypot administered by Frederic Perriot between August 24th, 2003 and February 24th, 2004, shown in Fig. 4.

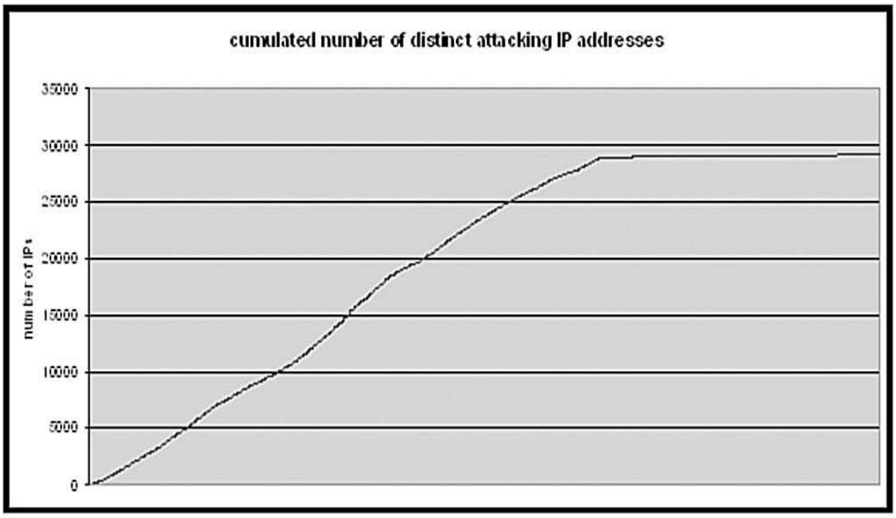


Figure 4: The cumulative number of Welchia attackers [3].

We will explore this example by photographing the data from Fig. 4.

$$data\_Welchia :=$$

$$\{\{1.1, 1000\}, \{2, 2333\}, \{3, 3500\}, \{4, 5000\}, \{5, 6833\}, \{6, 8000\},$$
$$\{7, 9333\}, \{8, 10500\}, \{9, 12000\}, \{10, 14000\}, \{11, 16333\},$$
$$\{12, 18167\}, \{13, 19667\}, \{14, 21000\}, \{15, 22667\}, \{16, 23667\},$$
$$\{17, 25000\}, \{18, 26333\}, \{19, 27500\}, \{20, 28333\}, \{21, 29333\},$$
$$\{22, 29500\}, \{23, 29500\}, \{24, 29500\}, \{25, 29500\}, \{26, 29500\},$$
$$\{27, 29500\}, \{28, 29500\}, \{29, 29500\}, \{30, 29500\}, \{31, 29667\},$$
$$\{32, 29667\}\}$$

The fitted model is given by

$$\lambda = -0.1;\ \ \beta = 0.0385232;\ \ \theta = 0.602142;\ \ \omega = 30000.$$

We receive an impressive result when approximating these data, see Fig. 5.

**Example 2.** Here we will present a new analysis of Conficker propagation in 2008 and we explore the Network Telescope project's daily dataset [4], [5] collected on November 21, 2008.

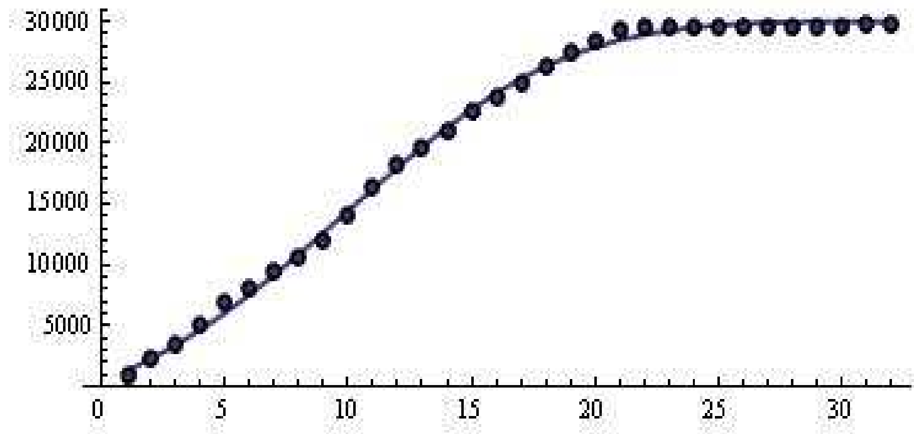


Figure 5: The fitted model $M^*(t)$.

We analyze the following data

$$data\_Conficker :=$$

$$\{\{0.1, 10\}, \{1, 150\}, \{2, 300\}, \{3, 600\}, \{4, 2500\}, \{5, 5000\},$$

$$\{6, 7500\}, \{7, 13000\}, \{8, 19000\}, \{9, 25000\}, \{10, 31000\},$$

$$\{11, 37000\}, \{12, 44000\}, \{13, 52000\}, \{14, 58000\}, \{15, 66000\},$$

$$\{16, 74000\}, \{17, 81000\}, \{18, 86000\}, \{19, 89000\}, \{20, 92000\},$$

$$\{21, 92500\}\}$$

The model (4) for $\omega = 93000$; $\lambda = -0.1$; $\beta = 0.00547243$ and $\theta = 0.757525$ is visualized on Fig. 6.

**Example 3.** Number of users attacked by Trojan-Ransom malware

We will study how it can be modelled data in [6] for the number of users attacked by Trojan-Ransom malware (Q4 2014 - Q3 2015).

The cumulative data is:

$$Number\_of\_users\_attacked\_by\_Trojan - Ransom\_malware$$

$$\_(Q4\_2014 - Q3\_2015)\_data :=$$

$$\{\{1, 128132\}, \{2, 278706\}, \{3, 543089\}, \{4, 880294\}\}$$

The fitted model $M^*(t) = \omega M(t)$ for $\omega = 1150000$; $\lambda = -0.04$; $\beta = 0.0731211$ and $\theta = 0.872133$ is visualized on Fig. 7.

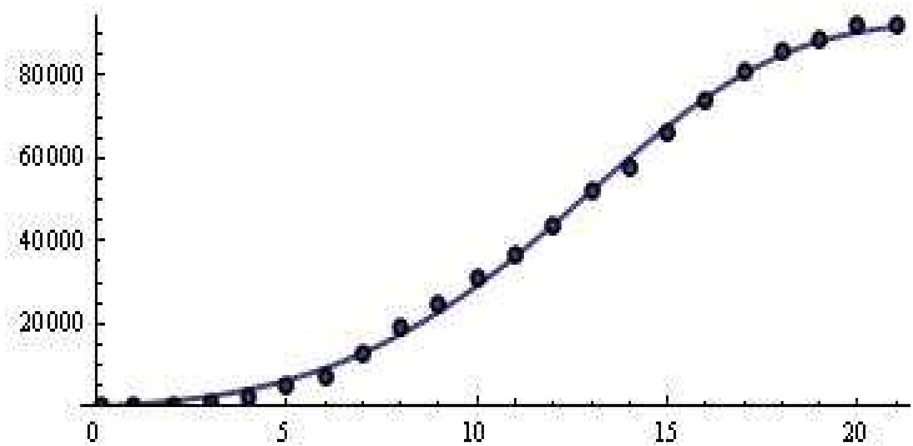


Figure 6: The fitted model $M^*(t)$.

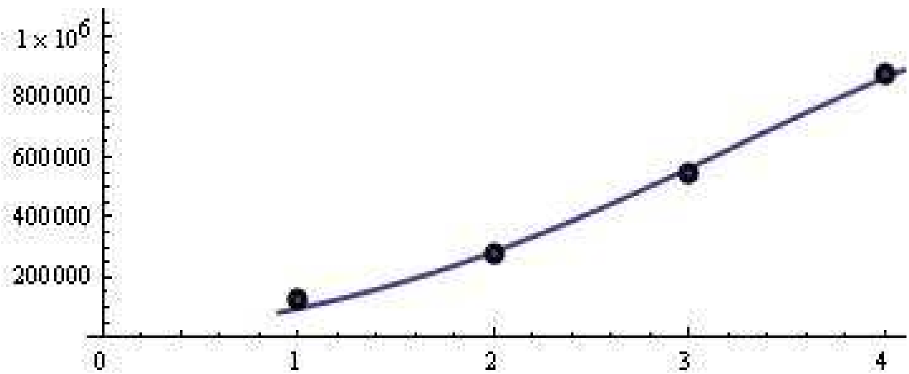


Figure 7: The fitted model $M^*(t)$.

## 3. CONCLUDING REMARKS

Finally, we note that the studied model produces extremely good results, generally when approximating specific "cumulative data" from Computer Viruses Propagation, Debugging and Test theory.

For other approximation and modelling results, see [7]–[29].

We hope that the results will be useful for specialists in this scientific area.

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