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ASSOCIATED LEGENDRE POLYNOMIALS AS CORRECTIONS IN THE CLASSICAL LIENARD SYSTEM

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ABSTRACT: In this article we consider a new extended Lienard–type differential system. The effects on the dynamics with "corrections" of the Associated Legendre polynomials (ALP) $L_n^2 n$ are studied.

Numerical examples, illustrating our results using *CAS MATHEMATICA* are given.

Key Words: Lienard system, Associated Legendre polynomial, level curves

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1. INTRODUCTION

A great number of mathematical models of physical systems give rise to differential equation of the Lienard's–type [1].

Various modifications of this model have been proposed and studied by a number of researchers (see for example [2]–[8]).

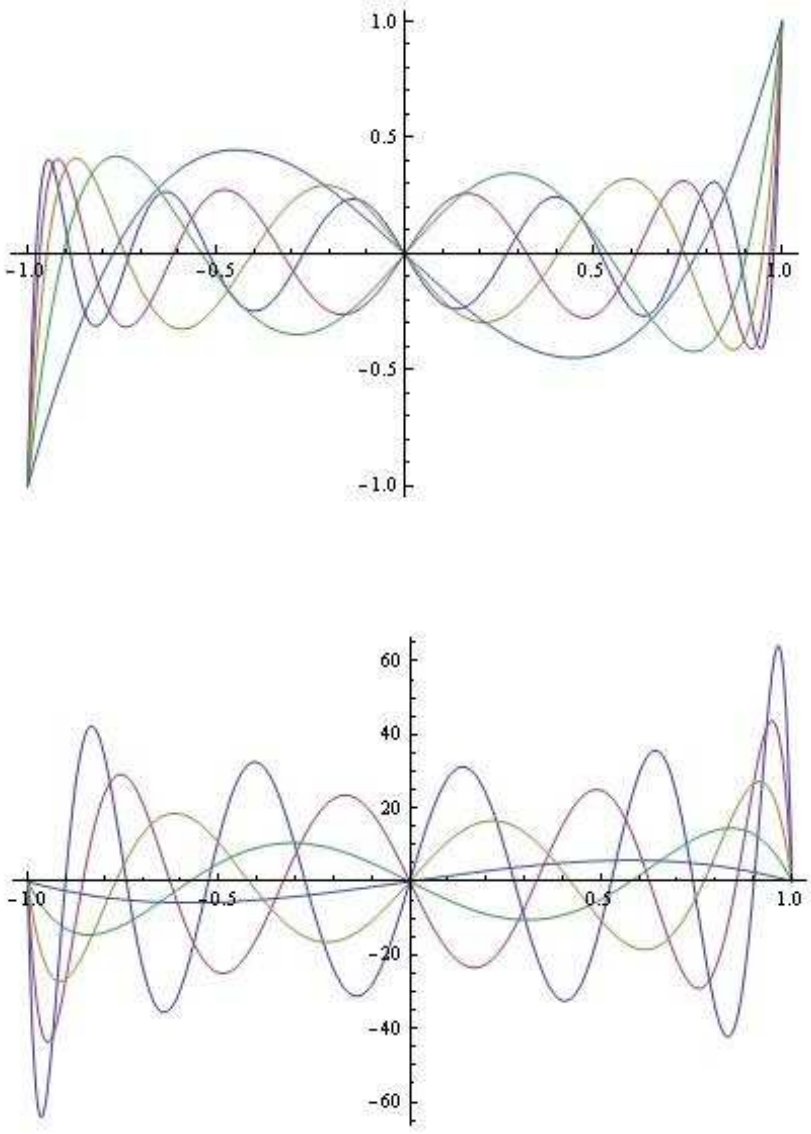

Figure 1: a) Legendre polynomials $L_n(x)$ for $n = 3, 5, 7, 9, 11$; b) Associated Legendre polynomials $L_n^2(x)$ for $n = 3, 5, 7, 9, 11$.

Some specific classes of generalized polynomial Lienard differential systems with additional "polynomial intervention factors" can be found in [9]–[22].

Research on the dynamics of Lienard–type differential systems with "associated corrections to the known orthogonal polynomials" is also an interesting topic that have plan to include in our future developments.

Without going into details, we will consider only one particular case.

2. ASSOCIATED LEGENDRE POLYNOMIALS AS CORRECTION FACTORS IN THE LIENARD DIFFERENTIAL SYSTEM

We will mention only one of the directions - study of the class of Lienard polynomial systems of the type

$$\begin{cases} \dfrac{dx}{dt} &= y \\ \\ \dfrac{dy}{dt} &= Poly(x) + \epsilon f(x) y \end{cases} \tag{1}$$

where $0 \leq \epsilon < 1$; $f(x)$ is a specially chosen polynomial, and $Poly(x)$ coincides with *Associated Legendre polynomial* $L_n^2(x)$.

For example for $L_n^2(x)$ we have (see Fig. 1b).

$$\begin{aligned} L_3^2(x) &= 15x - 15x^3 \\ L_5^2(x) &= \tfrac{105}{2}\left(-3x^5 + 4x^3 - x\right) \\ L_7^2(x) &= \tfrac{63}{8}\left(-143x^7 + 253x^5 - 125x^3 + 15x\right) \\ L_9^2(x) &= \tfrac{495}{16}\left(-221x^9 + 494x^7 - 364x^5 + 98x^3 - 7x\right) \\ L_{11}^2(x) &= \tfrac{2145}{128}\left(-2261x^{11} + 6137x^9 - 6018x^7 + 2562x^5 - 441x^3 + 21x\right) \end{aligned}$$

The solutions of the system (1) with $Poly(x) = L_9^2(x)$ and

$$f(x) = x - x^3 + x^5 - \frac{1}{7}x^7$$

for $\epsilon = 0.001; x_0 = 0.35, y_0 = 0.35$ is depicted on Fig. 2.

2.1. THE LEVEL CURVES

The case a) $Poly_i(x)$ coincides with $L_5^2(x)$

The Hamiltonian of system (1) ($\epsilon = 0$) is

$$H(x,y) = \frac{y^2}{2} + \frac{105}{4}x^6 - \frac{105}{2}x^4 + \frac{105}{4}x^2.$$

The level curves $L_{h_i} = \{H(x,y) = h_i\}$ are depicted at Fig. 3.

The case b) $Poly_i(x)$ coincides with $L_7^2(x)$

The Hamiltonian of system (1) ($\epsilon = 0$) is

$$H(x,y) = \frac{y^2}{2} + \frac{9009}{64}x^8 - \frac{15939}{48}x^6 + \frac{7875}{32}x^4 - \frac{945}{16}x^2.$$

The level curves $L_{h_i} = \{H(x,y) = h_i\}$ are depicted at Fig. 4.

Consider the following model in the light of Zeeman's approach:

$$\begin{cases} \frac{dx}{dt} &= c(F(x) - y) \\ \frac{dy}{dt} &= \frac{1}{c}x \end{cases} \tag{2}$$

with $c > 0$ and

$$F(x) = \frac{495}{16}\left(-221x^9 + 494x^7 - 364x^5 + 98x^3 - px\right)$$

The catastrophe surfaces $(x, y, p) = F(x) - y$ $(p = 20, 30, 50)$ for the model is depicted on Fig. 5.

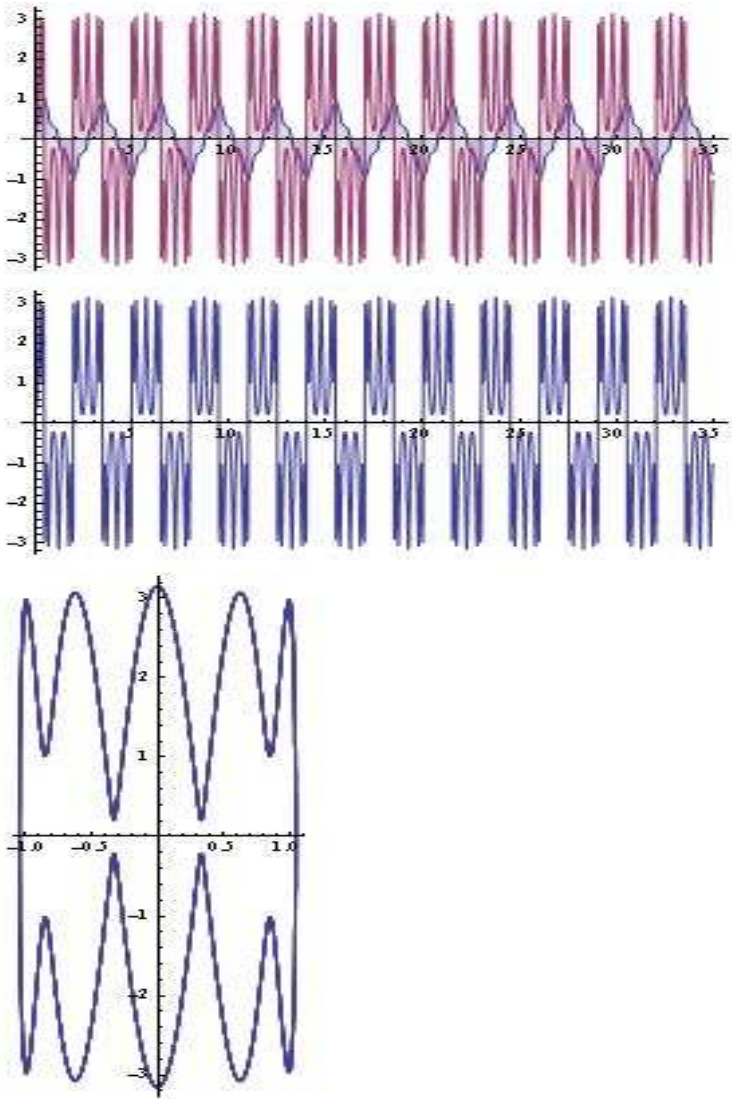

Figure 2: a) The solutions of the differential system (1); b) y-component of solution; c) The portrait.

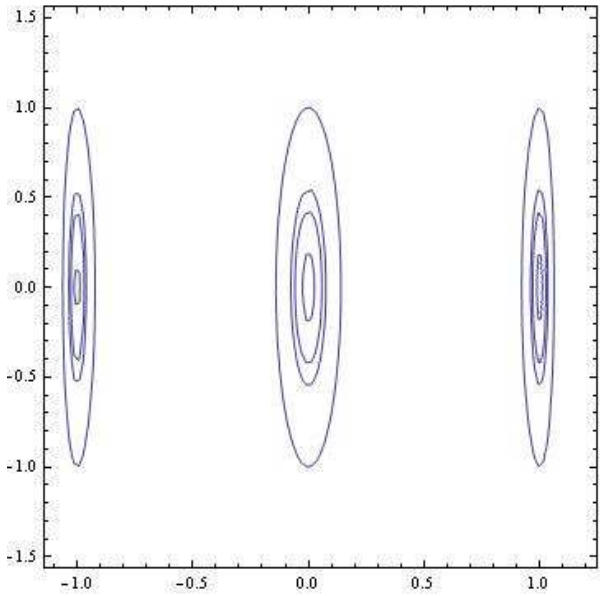

Figure 3: Level curves (the case a).

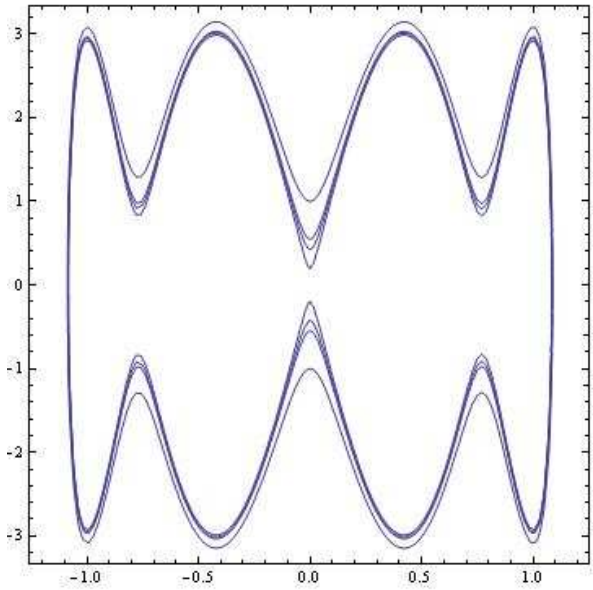

Figure 4: Level curves (the case b).

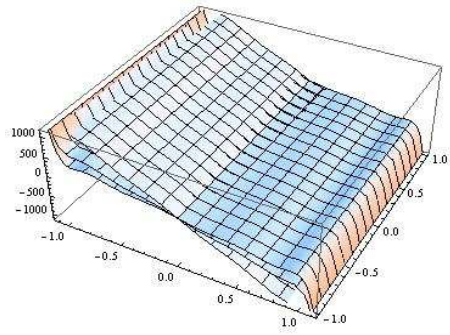

Figure 5: The catastrophe surfaces in the light of Zeeman considerations.

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