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NEW EXTENDED BASED ON GENERALIZATION OF HARRIS ALGORITHM

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ABSTRACT: In this note we develop new extended algorithm which generalize Harris algorithm [1], [11]. Our new computational process demonstrates benefits as superior speed and correct results obtained through so-called "hybrid" extended algorithm. We will explicitly note that algorithm in this article is first hybrid extended algorithm known in literature.

AMS Subject Classification: 11A05, 68W01
Key Words: extended Euclidean algorithm, Harris algorithm, hybrid extended algorithm

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1. INTRODUCTION

We briefly set the task here to searching and obtaining integer numbers x and y such that $x * a + y * b = Greatest\ Common\ Divisor\ (gcd)$ of a and b, where a and b are preliminary given natural numbers. For the many aspects of Euclidean algorithms, see [1]–[9] and [31]–[47]. Our new approach to this and other similar tasks is concretized in [10]–[30].

For testing purposes we will use the following computer: processor – Intel(R) Core(TM) i7-6700HQ CPU 2.60GHz, 2592 Mhz, 4 Core(s), 8 Logical Processor(s), RAM 16 GB, Microsoft Windows 10 Enterprise x64, Microsoft Visual C# 2017 x64.

2. MAIN RESULTS

We present new extended which generalizes Harris algorithm:

Algorithm 1.

```
x1 = 1; x2 = 0; y1 = 0; y2 = 1;
int g = 0;
if ((a & 1) == 0 && (b & 1) == 0)
do { a >>= 1; b >>= 1; g++; }
while ((a & 1) == 0 && (b & 1) == 0);
u = a; v = b;
while ((u & 1) == 0)
{
u >>= 1;
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
}
while ((v & 1) == 0)
{
v >>= 1;
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
}
if (u > v) do
{
q = u / v; u %= v;
```

```
if (u < 1) { x = y1; y = y2; gcd = v << g; break; }
x1 -= q * y1; x2 -= q * y2;
if ((u & 1) == 0)
{
do
{
u >>= 1;
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
} while ((u & 1) == 0);
if (u == 1) { x = x1; y = x2; gcd = u << g; break; }
}
else
{
if (u == 1) { x = x1; y = x2; gcd = u << g; break; }
u = v - u; x1 = y1 - x1; x2 = y2 - x2;
do
{
u >>= 1;
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
} while ((u & 1) == 0);
if (u == 1) { x = x1; y = x2; gcd = u << g; break; }
}
q = v / u; v %= u;
if (v < 1) { x = x1; y = x2; gcd = u < < g; break; }
y1 -= q * x1; y2 -= q * x2;
if ((v & 1) == 0)
{
do
{
v >>= 1;
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
} while ((v & 1) == 0);
```

```
if (v == 1) { x = y1; y = y2; gcd = v << g; break; }
}
else
{
if (v == 1) { x = y1; y = y2; gcd = v << g; break; }
v = u - v; y1 = x1 - y1; y2 = x2 - y2;
do
{
v >>= 1;
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
} while ((v & 1) == 0);
if (v == 1) { x = y1; y = y2; gcd = v << g; break; }
}
}
while (true);
else do
{
q = v / u; v %= u;
if (v < 1) { x = x1; y = x2; gcd = u << g; break; }
y1 -= q * x1; y2 -= q * x2;
if ((v & 1) == 0)
{
do
{
v >>= 1;
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
} while ((v & 1) == 0);
if (v == 1) { x = y1; y = y2; gcd = v << g; break; }
}
else
{
if (v == 1) { x = y1; y = y2; gcd = v << g; break; }
v = u - v; y1 = x1 - y1; y2 = x2 - y2;
```

```
do
{
v >>= 1;
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
} while ((v & 1) == 0);
if (v == 1) { x = y1; y = y2; gcd = v << g; break; }
}
q = u / v; u %= v;
if (u < 1) { x = y1; y = y2; gcd = v << g; break; }
x1 -= q * y1; x2 -= q * y2;
if ((u & 1) == 0)
{
do
{
u >>= 1;
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
} while ((u & 1) == 0);
if (u == 1) { x = x1; y = x2; gcd = u << g; break; }
}
else
{
if (u == 1) { x = x1; y = x2; gcd = u << g; break; }
u = v - u; x1 = y1 - x1; x2 = y2 - x2;
do
{
u >>= 1;
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
} while ((u & 1) == 0);
if (u == 1) { x = x1; y = x2; gcd = u << g; break; }
}
}
while (true);
```

as well as its recursive version as

Algorithm 2.

```
static long Euclid(long u, long v, long a, long b,
ref long x, ref long y, long x1, long x2, long y1, long y2, int g)
{
long q;
if (u > v)
{
q = u / v; u %= v;
if (u < 1) { x = y1; y = y2; return v << g; }
x1 -= q * y1; x2 -= q * y2;
if ((u & 1) == 0)
{
if (u == 1) { x = x1; y = x2; return u << g; }
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
return Euclid(u >> 1, v, a, b, ref x, ref y, x1, x2, y1, y2, g);
}
else
{
if (u == 1) { x = x1; y = x2; return u << g; }
u = v - u; x1 = y1 - x1; x2 = y2 - x2;
if ((u & 1) == 0)
{
if (u == 1) { x = x1; y = x2; return u << g; }
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
return Euclid(u >> 1, v, a, b, ref x, ref y, x1, x2, y1, y2, g);
}
}
}
else
{
```

```
q = v / u; v %= u;
if (v < 1) { x = x1; y = x2; return u << g; }
y1 -= q * x1; y2 -= q * x2;
if ((v & 1) == 0)
{
if (v == 1) { x = y1; y = y2; return v << g; }
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
return Euclid(u, v >> 1, a, b, ref x, ref y, x1, x2, y1, y2, g);
}
else
{
if (v == 1) { x = y1; y = y2; return v << g; }
v = u - v; y1 = x1 - y1; y2 = x2 - y2;
if ((v & 1) == 0)
{
if (v == 1) { x = y1; y = y2; return v << g; }
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
return Euclid(u, v >> 1, a, b, ref x, ref y, x1, x2, y1, y2, g);
}
if (v == 1) { x = y1; y = y2; return v << g; }
}
}
return Euclid(u, v, a, b, ref x, ref y, x1, x2, y1, y2, g);
}
```

The recursive function can be called by:

```
x1 = 1; x2 = 0; y1 = 0; y2 = 1;
int g = 0;
if ((a & 1) == 0 && (b & 1) == 0)
do { a >>= 1; b >>= 1; g++; }
while ((a & 1) == 0 && (b & 1) == 0);
```

```
u = a; v = b;
while ((u & 1) == 0)
{
u >>= 1;
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
}
while ((v & 1) == 0)
{
v >>= 1;
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
}
gcd = Euclid(u, v, a, b, ref x, ref y, x1, x2, y1, y2, g);
```

Numerical Example.

For testing purposes of Algorithms 1 and 2 we will use the following main function:

```
long a, b, gcd, d1 = 0, x = 0, y = 0;
long x1, x2, y1, y2, q, u, v;
for (int i = 1; i < 100000001; i++) { a = i; b = 200000002 - i;
//here are placed the source code of algorithm 1 and
//calling of recursive algorithm 2
d1 += gcd;
}
Console.WriteLine(d1);
```

CPU time results are:

CPU time of Algorithm 1 is: **48.056 seconds.**

CPU time of Algorithm 2 is: **86.152 seconds.**

3. CONCLUSION

We claim new extended algorithm. Numerical experiments give us undoubtable reason for its theoretical and practical importance.

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