Communications in Applied Analysis, **26**, No. 1 (2022), 9-18 ISSN: 1083-2564

ON SOME UNDERSTUDIED MODELS WITH APPLICATIONS IN THE FIELD OF DEBUGGING THEORY

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ABSTRACT: Following the ideas given in [4]–[6] in this article, we analyze some understudied models, such as the Almalki and Bakouch models. It is shown how these models can be modified in view of their possible application for approximation of data from a real test of software modules and platforms.

Some numerical examples, using *CAS MATHEMATICA* are also given.

AMS Subject Classification: 41A46

Key Words: modified reduced Weibull c.d.f. based on the model by Almalki, modified logistic model by Bakouch, root mean square error

Received: December 19, 2021 ; **Accepted:** February 22, 2022 ;
Published: March 2, 2022 **doi:** 10.12732/caa.v26i1.2
Dynamic Publishers, Inc., Acad. Publishers, Ltd. http://www.acadsol.eu/caa

1. A LOOK AT THE MODEL BY ALMALKI [?]

Consider the following modified reduced Weibull c.d.f. based on the model by Almalki [2]:

$$f_1(t) = A - e^{-\sqrt{kt}-\sqrt{kt}e^{kt}} \tag{1}$$

for $t \geq 0$, $k, A > 0$ and

$$f_1(0) = A - 1; \quad \lim_{t \to +\infty} f_1(t) = A.$$

Suppose that in his experimental work, the researcher observes a series of experimental data that are located at a slightly lower level than the asymptotic level A guaranteed by the fixed base model (1).

Consider the new model

$$f_2(t) = A - e^{-\sqrt{\frac{kt}{1+kt}} - \sqrt{\frac{kt}{1+kt}} e^{\frac{kt}{1+kt}}}, \tag{2}$$

for which

$$f_2(0) = A - 1; \quad \lim_{t \to +\infty} f_2(t) = A - e^{-1-e} := B.$$

The models (1) and (2) are depicted on Fig. 1 for some values of parameters k and A.

1.1. NUMERICAL EXAMPLES

Here we will demonstrate the advantage of model (2) over model (1) in approximating data, for example in the field of debugging theory.

For example, we will use the following (abbreviated) test data (t_i, y_i) from a software platform:

$$DataFailure1 := \{\{0.005, 0.4\}, \{0.05, 0.77\}, \{0.1, 0.89\}, \{0.8, 0.99\},$$

$$\{0.9, 0.995\}, \{1, 1.023\}\}$$

From the attached graph (Fig. 2) it is seen that the approximation of the data with model (2) is better - in the root mean square sense.

Let

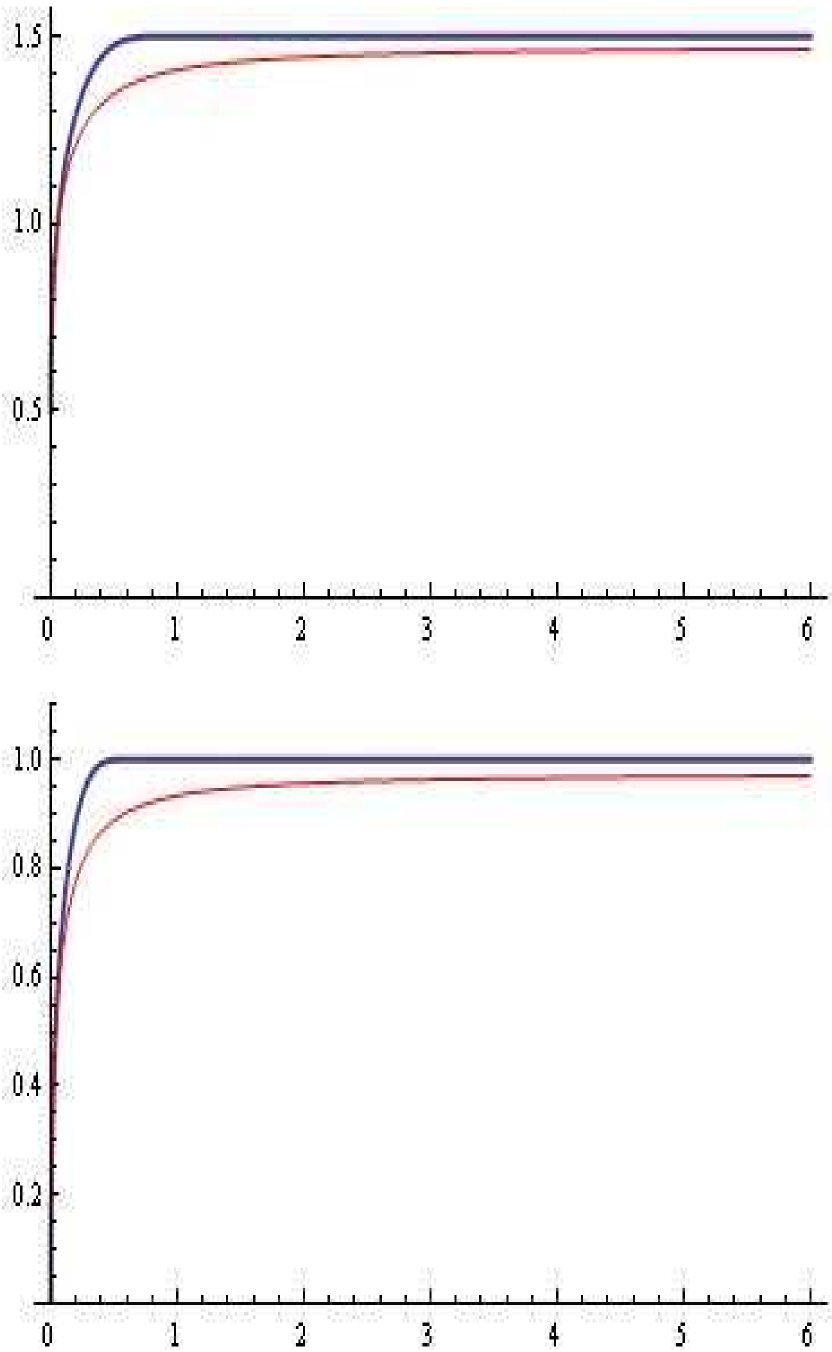

Figure 1: The functions $f_1(t)$ and $f_2(t)$ for a) $A = 1.5, k = 2,\ B = 1.47572$; b) $A = 1, k = 3,\ B = 0.975724$.

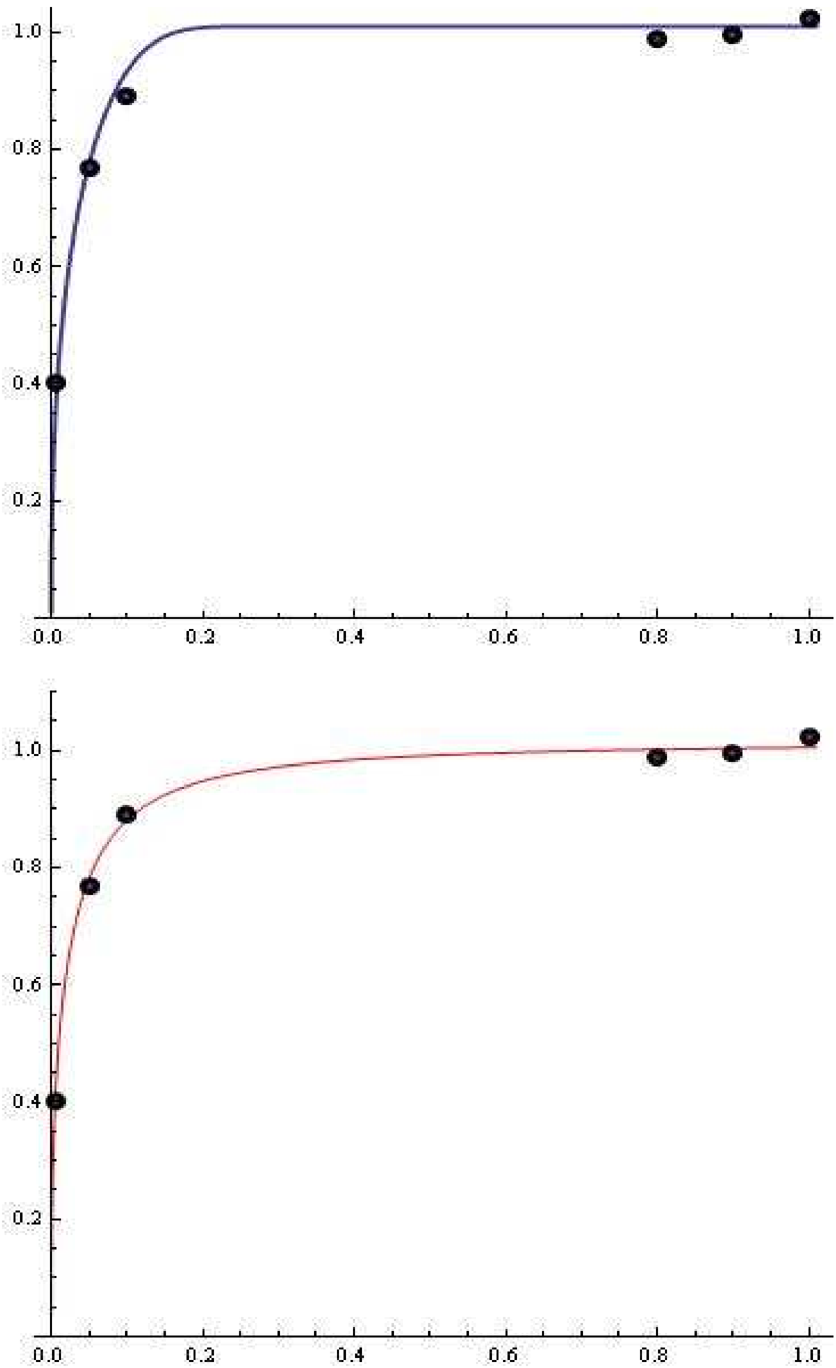

Figure 2: The fitted models: a) $f_1(t)$ for $A = 1.01013$; $k = 7.12214$; b) $f_2(t)$ for $A = 1.04258$; $k = 9.36736$ (for the "dataFailure1").

$$\epsilon_j = \sum_{i=1}^{N} \left(f_j(t_i) - y_i\right)^2, \;\; j = 1, 2.$$

Evidently, for the error we have: $\epsilon_1 = 0.00764309$ for the fitted model (1) and $\epsilon_2 = 0.00050394$ for the model (2).

Remark.

The user of such methods can use in practice the following simplified model

$$A(t) = C\left(1 - e^{-\sqrt{kt} - \sqrt{kt}e^{kt}}\right)$$

and its analogue

$$A_1(t) = C_1\left(1 - e^{-\sqrt{k_1 t} - \sqrt{k_1 t}e^{\frac{k_1 t}{1+k_1 t}}}\right).$$

We will conduct an experiment on the following dataset

$DataFailure2 := \{\{0.01, 1.6\}, \{0.02, 10\}, \{0.03, 20\}, \{0.05, 30\},$

$\{0.09, 40\}, \{0.12, 60\}, \{0.18, 78\}, \{0.21, 80\}, \{0.24, 82\},$

$\{0.27, 82\}, \{0.3, 82\}, \{0.32, 80\}, \{0.35, 80\}, \{0.41, 80\},$

$\{0.44, 100\}, \{0.49, 100\}, \{0.53, 100\}, \{0.59, 110\}, \{0.62, 110\},$

$\{0.67, 113\}, \{0.7, 114\}, \{0.75, 110\}, \{0.8, 110\}, \{0.9, 110\},$

$\{1, 118\}, \{1.3, 126\}, \{1.4, 125\}, \{1.5, 126\}, \{1.9, 123\},$

$\{2, 130\}, \{3, 130\}, \{4, 130\}\}$

An approximation of the data is visualized in Figure 3.

From the specifics of the data, the user can orient himself and judge which of the proposed models gives better results.

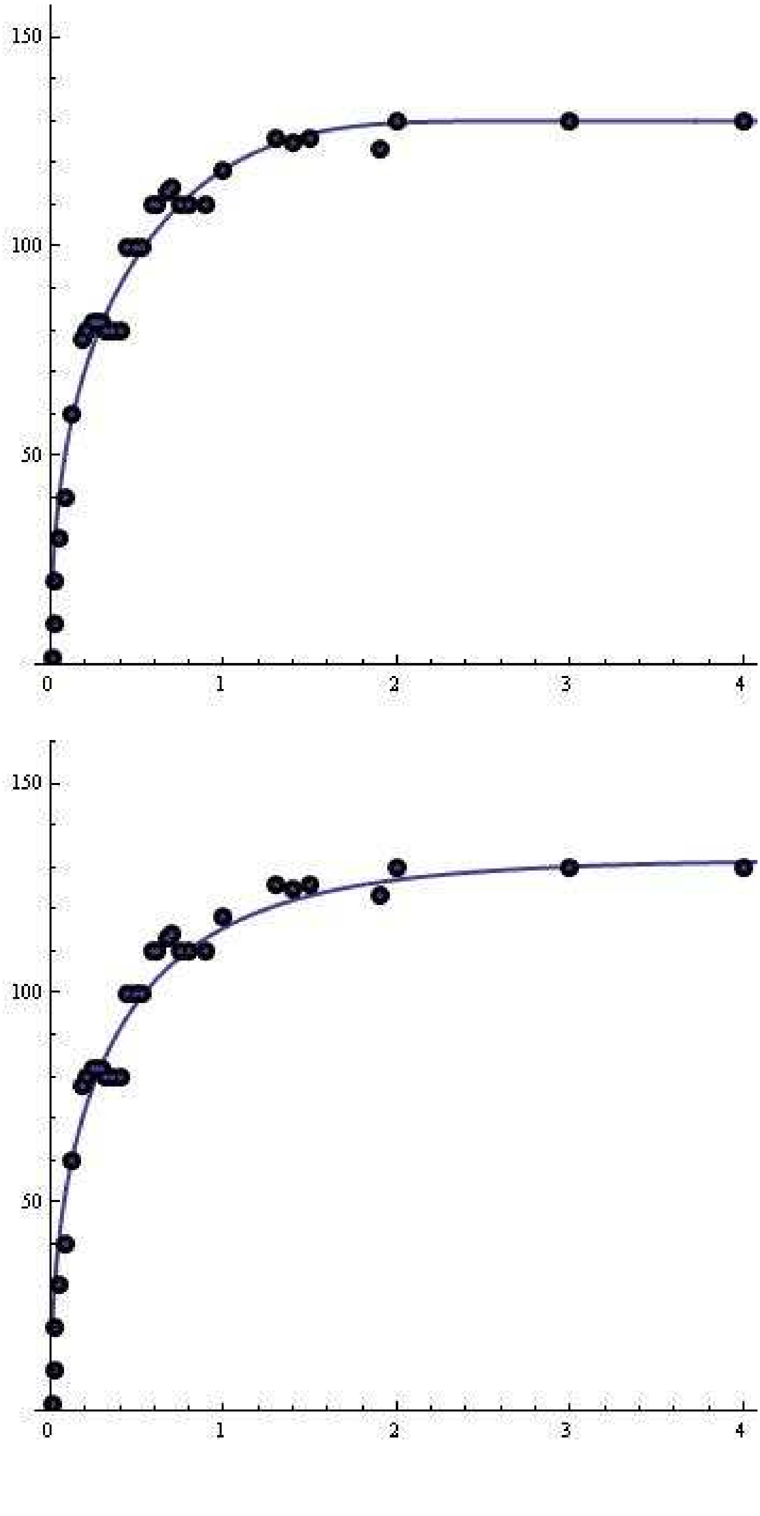

Figure 3: The fitted models: a) $A(t)$ for $C = 130$; $k = 0.669313$; b) $A_1(t)$ for $C_1 = 132$; $k_1 = 0.690415$ (for the "dataFailure2").

2. A LOOK AT THE MODEL BY BAKOUCH [?].

Consider the following modified logistic model by Bakouch [3]:

$$g_1(t) = \frac{a\left(1 - e^{-kt}\right)}{a + e^{-kt}} \tag{3}$$

for $t \geq 0,\ k, a > 0$

and

$$g_1(0) = 0; \quad \lim_{t\to+\infty} g_1(t) = 1.$$

Suppose that the researcher observes a series of experimental data that are located at a slightly lower level than the asymptotic level 1 guaranteed by the fixed base model (3).

Consider the new model

$$g_2(t) = \frac{a\left(1 - e^{-kt}\right)}{a + e^{-\frac{kt}{1+kt}}}. \tag{4}$$

for which

$$g_2(0) = 0; \quad \lim_{t\to+\infty} g_2(t) = \frac{a}{a + e^{-1}} := B.$$

The models (3) and (4) are depicted on Fig. 4 for some values of parameters k and a.

Concluding Remark.

In conclusion, we will note that with the proposed methodology (see, [4]–[6]), the researcher can modify other known (more advanced) models for the needs of his specific research.

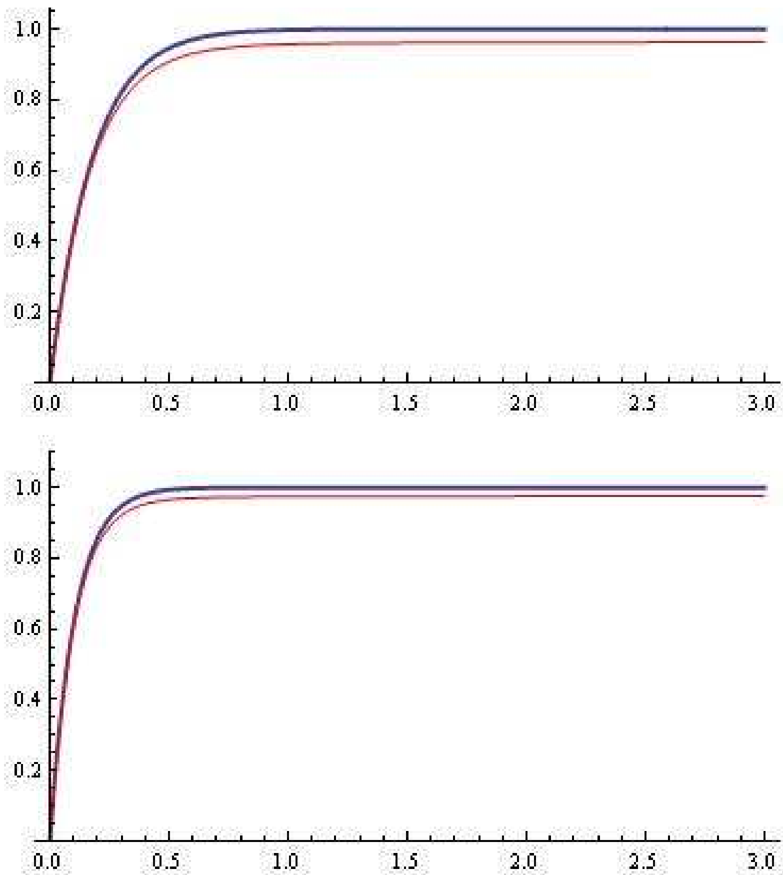

Figure 4: The functions $g_1(t)$ and $g_2(t)$ for a) $a = 10, k = 6$; b) $a = 15, k = 10$.

ACKNOWLEDGMENTS

This work has been accomplished with the financial support by the Grant No BG05M2OP001-1.001-0003, financed by the Science and Education for Smart Growth Operational Program (2014-2020) and co-financed by the European Union through the European structural and Investment funds.

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