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AN OVERVIEW OF A SEEMINGLY MORE SOPHISTICATED GROWTH FUNCTIONS: ASSOCIATED PIECEWISE SMOOTH FUNCTIONS. APPLICATIONS

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ABSTRACT: Following the ideas given in [1]–[3], in this article we study more sophisticated growth models of the type: $h_1(t) = A - e^{1-kt-e^{kt}}$ and their "hypothetical piecewise smooth functions". Some numerical examples, using *CAS MATHEMATICA* are also given.

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1. MAIN RESULTS

Growth curves are found in a wide range of disciplines, such as population dynamics, bacterial growth, population ecology, plant biology, chemistry, statistics and medical science.

Suppose for a moment that in his experimental work, the researcher has the confidence that the data obtained by him can be closely related to the established practice of modeling similar data - a seemingly more sophisticated family of the type:

$$h_1(t) = A - e^{1-kt-e^{kt}}, \tag{1}$$

for which

$$h_1(0) = A - 1; \quad \lim_{t \to +\infty} h_1(t) = A.$$

Without community constraint, suppose $A = \frac{3}{2}$.

Assume also that for a long period of time the researcher observes a series of experimental data that are located at a slightly lower level than the asymptotic level A guaranteed by the fixed base model (1). (This is a typical case of debugging and test theory!!!).

For example, let at the end of the experiment, the "grouped data" be at an approximately asymptotic level $A - e^{-e} \approx 1.43401$, the researcher can achieve a good approximation of the data using the function:

$$h_2(t) = A - e^{1-\frac{kt}{1+kt}-e^{\frac{kt}{1+kt}}}, \tag{2}$$

for which

$$h_2(0) = A - 1; \quad \lim_{t \to +\infty} h_2(t) = A - e^{-e} \approx 1.43401 := B.$$

It is easy to see that the hypothetical piecewise smooth growth model is of the form:

$$H(t) := \begin{cases} A - e^{1-kt-e^{kt}} := h_1(t), \ \ t < 0 \\ \\ A - 1, \ \ t = 0 \\ \\ A - e^{1-\frac{kt}{1+kt}-e^{\frac{kt}{1+kt}}} := h_2(t), \ \ t > 0. \end{cases} \tag{3}$$

Evidently,

$$h_1'(0) = h_2'(0).$$

The hypothetical piecewise smooth growth model $H(h_1(t), h_2(t))$ is depicted on Figures 1–2.

Of course, this is not just a simulation of the asymptotic level, which can be achieved by correcting the parameter A in the basic model (1), but more importantly - coverage and good approximation of data from the above scientific field, which are not characterized by a "rapid outbreak" in the initial period of the experiment (see Figures 1,2,4).

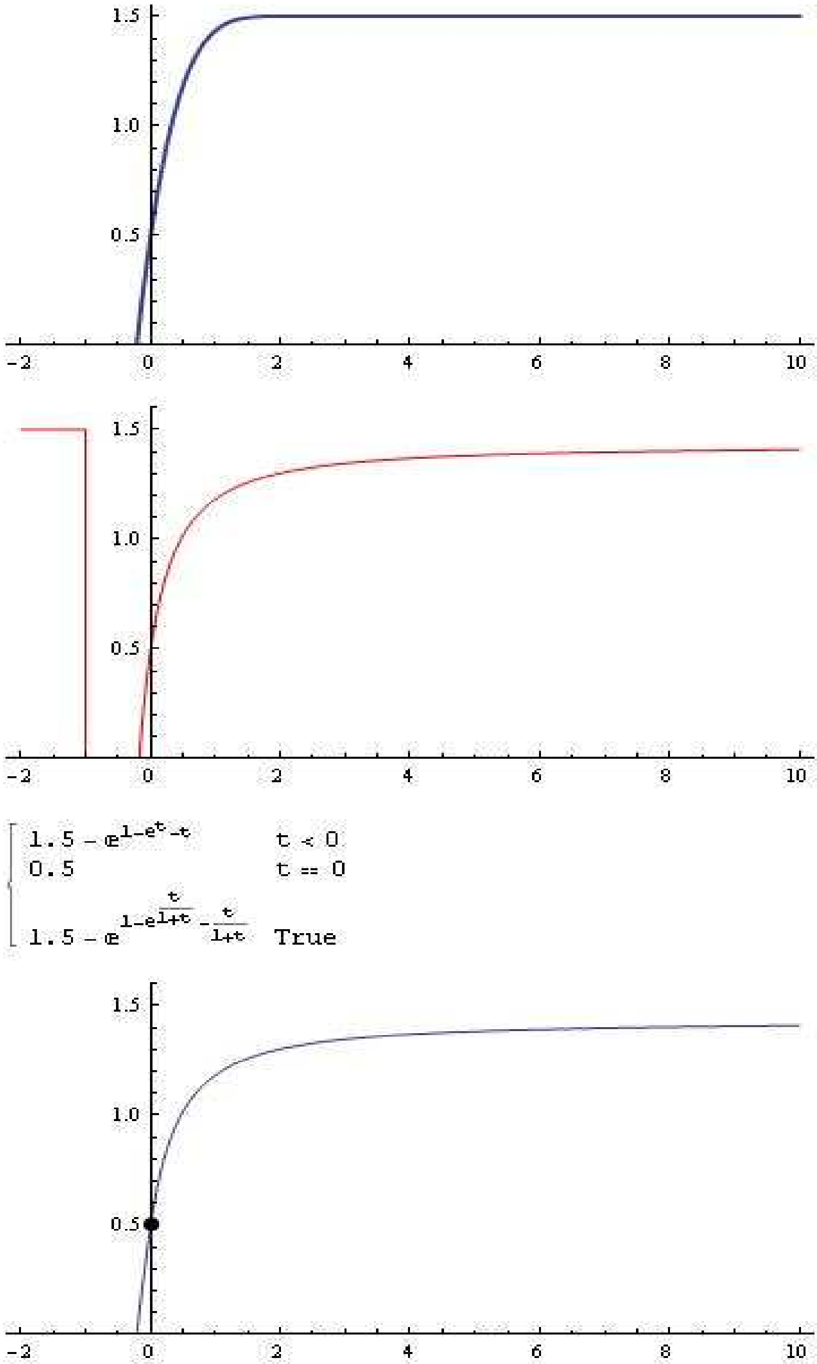

Figure 1: The functions $h_1(t)$, $h_2(t)$ and $H(h_1(t), h_2(t))$ for $A = 1.5$, $B = 1.43401$, $k = 1$.

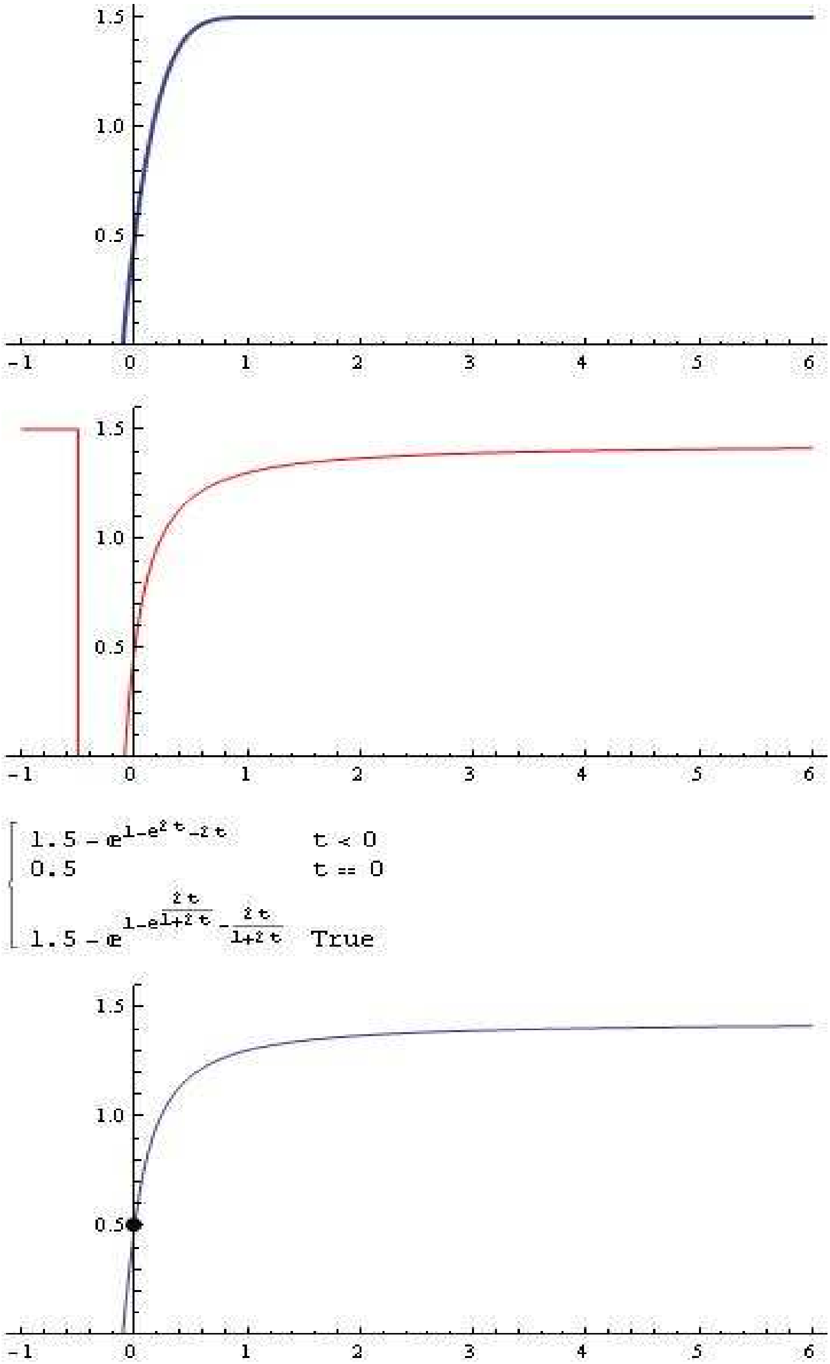

Figure 2: The functions $h_1(t)$, $h_2(t)$ and $H(h_1(t), h_2(t))$ for $A = 1.5$, $B = 1.43401$, $k = 2$.

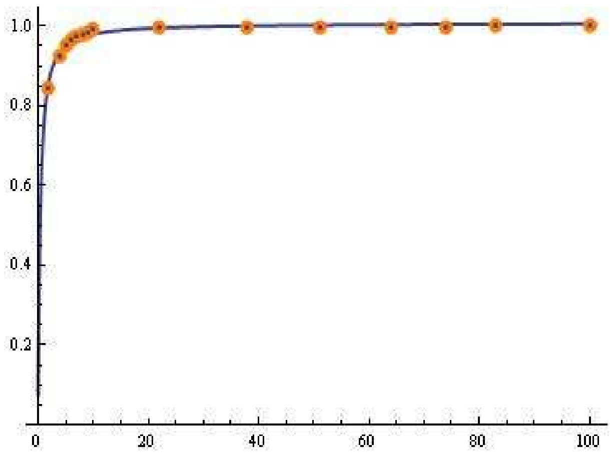

Figure 3: The fitted model $h_2(t)$ for $A = 1.07388;\ \ k = 0.921153$ (for the "dataStorm").

2. SOME APPLICATIONS

We will explicitly note that the sigmoidal function $h_2(t)$ gives very good results in approximating data sets in the fields of debugging theory and the spread of computer viruses.

We will illustrate what has been said with an appropriate example.

Example 1. For the

$$dataStorm := \{\{1.8, 0.843\}, \{4, 0.926\}, \{5, 0.954\}, \{6, 0.967\},$$

$$\{7, 0.976\}, \{8, 0.981\}, \{9, 0.985\}, \{10, 0.991\}, \{22, 0.995\}, \{38, 0.997\},$$

$$\{51, 0.998\}, \{64, 0.9985\}, \{74, 0.999\}, \{83, 1\}, \{100, 1\}\}$$

the fitted model

$$h_2(t) = A - e^{1-\frac{kt}{1+kt}-e^{\frac{kt}{1+kt}}}$$

for $A = 1.07388;\ \ k = 0.921153$ is depicted on Fig. 3.

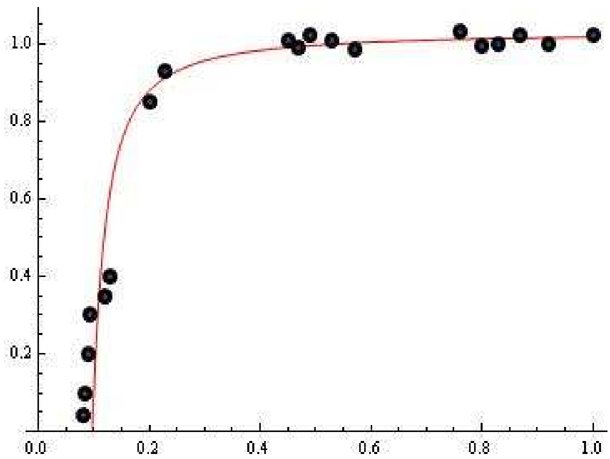

Figure 4: The fitted model $h_2^*(t)$ for $A = 1.1;\ k = 17;\ r = 0.1$ (for the "dataFailure").

Example 2. For the

$$DataFailure := \{\{0.08, 0.04\}, \{0.084, 0.1\}, \{0.089, 0.2\}, \{0.093, 0.3\},$$

$$\{0.12, 0.35\}, \{0.13, 0.4\}, \{0.2, 0.85\}, \{0.23, 0.93\}, \{0.45, 1.01\},$$

$$\{0.47, 0.99\}, \{0.49, 1.02\}, \{0.53, 1.01\}, \{0.57, 0.983\}, \{0.76, 1.03\},$$

$$\{0.8, 0.996\}, \{0.83, 0.998\}, \{0.87, 1.02\}, \{0.92, 1\}, \{1, 1.023\}\};$$

the fitted model

$$h_2^*(t) = A - e^{1 - \frac{k(t-r)}{1+k(t-r)} - e^{\frac{k(t-r)}{1+k(t-r)}}}$$

for $A = 1.1;\ k = 17;\ r = 0.1$ is depicted on Fig. 4.

The results are satisfactory. The presented approach can be used successfully in the analysis of grouped data.

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