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A LOOK AT THE NEW LOGARITHMIC TRANSFORMED ADAPTIVE G–FAMILIES: PROPERTIES AND APPLICATIONS

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ABSTRACT: In this article we consider new modified logarithmic transformed adaptive G– families (LTAGM). Some properties for special classes of the families (with "fractional linear correction" and "Sin–G correction) are studied. We study also the "saturation" in the Hausdorff sense for some special cases of the families. Numerical examples, illustrating our results using *CAS MATHEMATICA* are given.

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1. INTRODUCTION

In [1] the authors proposed the following logarithmic transformed cumulative distribution

$$M_1(t) = 1 - \frac{\ln\left(1+\delta-\delta^{G(t)}\right)}{\ln\delta}, \tag{1}$$

where $G(t)$ is the baseline (cdf).

In particular, for $G(t) = e^{-\lambda t^{-\alpha}}$ [1]:

$$M_2(t) = 1 - \frac{\ln\left(1+\delta-\delta^{e^{-\lambda t^{-\alpha}}}\right)}{\ln\delta} \tag{2}$$

for $t > 0,\ \alpha > 0,\ \lambda > 0,\ \delta > 0$.

Various modifications of this "powerful" class of functions have been proposed and studied by a number of researchers.

In [2] we study the family (2) for $\alpha = 1$ and $\lambda = \delta$, i.e.

$$M_3(t) = 1 - \frac{\ln\left(1+\delta-\delta^{e^{-\frac{\delta}{t}}}\right)}{\ln\delta} \tag{3}$$

and prove that for sufficiently small values of δ and $d \le \frac{1}{2}$ the following estimate for the "saturation" - d about Hausdorff metric [4] is valid:

$$d \approx \frac{\delta}{1+\ln\left(\ln\left(\frac{1}{\delta}\right)\right)}. \tag{4}$$

We bring to the reader's attention the following interesting modification [3]:

$$M_4(t) = 1 - \frac{\ln\left(1+\delta-\delta^{e^{-\left(\frac{\delta}{t}\right)^{\delta}}}\right)}{\ln\delta} \tag{5}$$

which can be considered as an adaptive function with applications to the Antenna–feeder Analysis.

Definition 1. [4] The Hausdorff distance (the H–distance) [4] $\rho(f,g)$ between two interval functions f,g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{ \sup_{A\in F(f)} \inf_{B\in F(g)} ||A-B||, \sup_{B\in F(g)} \inf_{A\in F(f)} ||A-B||\}, \tag{6}$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A=(t_A,x_A)$, $B=(t_B,x_B)$ in $\mathbb{R}^2$ is $||A-B|| = \max(|t_A-t_B|,|x_A-x_B|)$.

In this article we study some properties of the family (1) for a special choice of the "correction" - $G(t)$.

2. MAIN RESULTS

1. In this section we consider a modified family of *"adaptive functions"* with *"fractional linear correction"*, i.e.

$$M_5(t) = 1 - \frac{\ln\left(1+\delta-\delta^{\frac{at}{1+at}}\right)}{\ln\delta}, \quad t>0,\ \delta>0,\ a>>1. \tag{7}$$

Evidently, for $G(t) = \frac{at}{1+at}$ we have

$$G(0)=0; \quad \lim_{t\to\infty} G(t) = 1.$$

For the "saturation" - d in the Hausdorff sense to the horizontal asymptote using $M_5(t)$ we have

$$M_5(d) = 1-d. \tag{8}$$

The modified family $M_5(t)$ for

a) $\delta = 0.1,\ a=10,\ d=0.102931;$

b) $\delta = 0.01,\ a=35,\ d=0.0247729$

is plotted on Fig. 1.

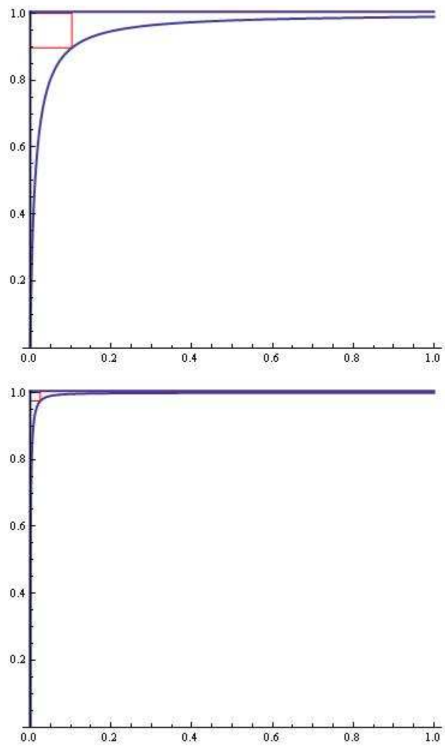

Figure 1: The function $M_5(t)$ for: a) $\delta = 0.1,\ a = 10,\ d = 0.102931$; b) $\delta = 0.01,\ a = 35,\ d = 0.0247729$.

A large proportion of the observed computer viruses are characterized by rapid growth over a relatively short period of time, after which gradual cumulative saturation usually occurs.

This was the reason why we proposed a new family of cumulative functions that has these properties.

Storm worm was one of the most biggest cyber threats of 2008.

$dataStorm := \{\{1.8, 0.843\}, \{4, 0.926\}, \{5, 0.954\}, \{6, 0.967\}, \{7, 0.976\},$

$\{8, 0.981\}, \{9, 0.985\}, \{10, 0.991\}, \{22, 0.995\}, \{38, 0.997\}, \{51, 0.998\},$

$\{64, 0.9985\}, \{74, 0.999\}, \{83, 1\}, \{100, 1\}\}$

For the "data_Storm"– normalized (see, [5] for some details) the fitted

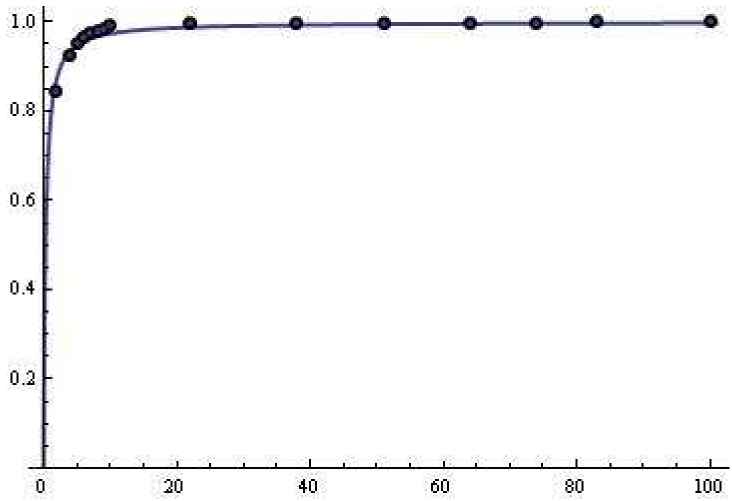

Figure 2: Epidemic data of Storm worm (normalized) fitted by $M_5(t)$ for $\delta = 0.152;\ a = 0.574136$.

model $M_5(t)$ for $\delta = 0.152;\ a = 0.574136$ is depicted on Fig. 2.

The following is valid

Proposition 1. Let $0 < \delta \leq 0.1$, $0 < d < \frac{1}{2}$. For sufficiently large values of a, for the "saturation" - d we have

$$d \approx \frac{\ln(0.06(1 + \frac{a}{\delta}))}{0.06(1 + \frac{a}{\delta})} := d^*. \tag{9}$$

Insofar as the proof is based on a technique proposed in [2], we will only note that from (8) it is easy to see that d is the only positive root of the nonlinear equation:

$$F(d) := \delta^d + \delta^{\frac{ad}{1+ad}} - 1 - \delta = 0. \tag{10}$$

Evidently, the function

$$H(d) := -1 + (1 + \frac{a}{\delta})d$$

approximates $F(d)$ with $d \to 0$ as $\mathcal{O}(d^2)$.

After a precise analysis we get the estimate (9).

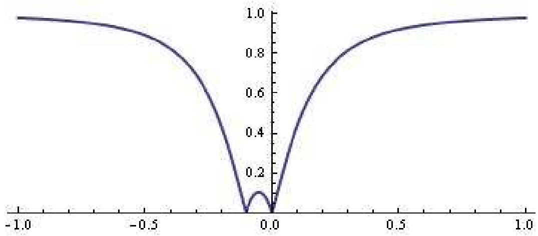

Figure 3: The model $M_6(t)$ for $\delta = 0.11,\ a = 0.5,\ b = 4.9,\ a_1 = 0.6;$ $t \in [-1, 1]$.

Some computational examples using relations (10) and (9) are presented in Table 1.

δ	a	d *computed by* (10)	d^* *computed by* (9)
0.01	20	0.0340131	0.03988
0.01	30	0.0270448	0.028842
0.01	35	0.0247729	0.0254565
0.05	300	0.0136417	0.016348
0.05	400	0.0117472	0.0128607
0.1	500	0.0143568	0.0190095
0.1	900	0.0106668	0.011650

Table 1: Bounds for d computed by (10) and (9) for various values of δ and a.

We will explicitly note that the estimate (9) may be useful for users due to the fact that the adaptation of this model in an arbitrary Computer Algebraic Calculation System presupposes the knowledge of an appropriate initial approximation for the root of the nonlinear equation (10), and, moreover, it is necessary double precision operation.

Remark. The reader may formulate other generalized G-classes, where G is a fractionally rational function.

As an example, we will define the following adaptive–G function, which finds application in the analysis of filter characteristics:

$$M_6(t) = 1 - \frac{\ln\left(1 + \delta - \delta^{\left|\frac{at+bt^2}{1+a_1t+bt^2}\right|}\right)}{\ln \delta}. \quad (11)$$

The model $M_6(t)$ for $\delta = 0.11$, $a = 0.5$, $b = 4.9$, $a_1 = 0.6$ is plotted on Fig. 3.

2. Consider the following modification of the model (1) with "correction" of type $G(t) = \sin(\frac{\pi}{2} \cdot \frac{at}{1+at})$. i.e.:

$$M_7(t) = 1 - \frac{\ln\left(1 + \delta - \delta^{\sin(\frac{\pi}{2} \cdot \frac{at}{1+at})}\right)}{\ln \delta}. \quad (12)$$

For the "saturation" - d in the Hausdorff sense to the horizontal asymptote using $M_7(t)$ we have

$$M_7(d) = 1 - d. \quad (13)$$

The model $M_7(t)$ for $\delta = 0.1$, $a = 40$ is plotted on Fig. 4.

From nonlinear equation (13) for the Hausdorff distance we find $d = 0.0316215$.

In some cases, model $M_7(t)$ achieves better saturation (in the Hausdorff sense) than $M_5(t)$.

The reader can obtain a corresponding grade of the type (9) using the result obtained in this article and we will omit it here.

Let $t = b\cos\theta + c$. Consider the function $|M_7(\theta)|$.

Then, for example, a typical antenna diagram using $|M_7(\theta)|$ for $\delta = 0.12$, $a = 0.17$, $b = -1.2$, $c = 0.005$ is depicted on Fig. 5.

3. CONCLUDING REMARKS

The reader can also define other modified surnames by making appropriate adjustments for the purposes of his research in various branches of scientific knowledge.

For some modelling and approximation problems, see [6]–[29].

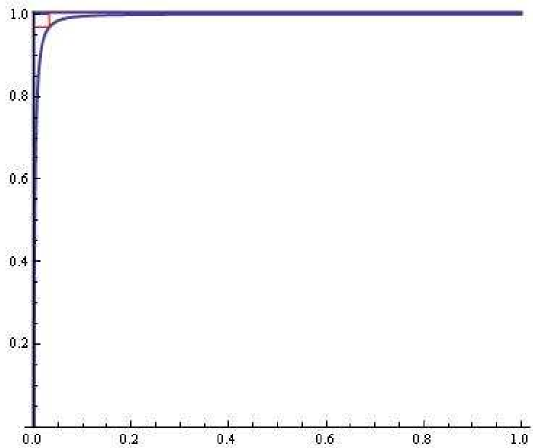

Figure 4: The model $M_7(t)$ for $\delta = 0.1$, $a = 40$; Hausdorff distance $d = 0.0316215$.

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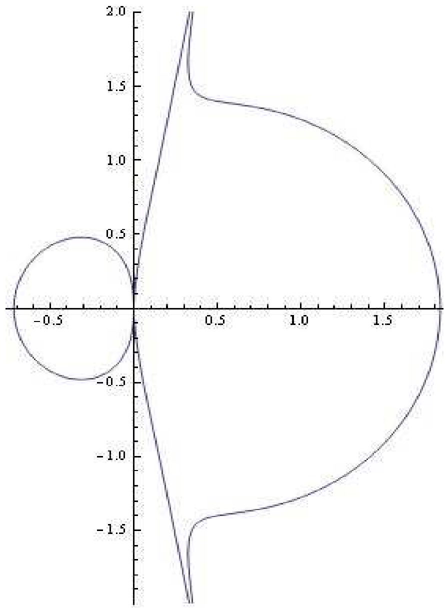

Figure 5: A typical antenna diagram using $|M_7(\theta)|$ for $\delta = 0.12$, $a = 0.17$, $b = -1.2$, $c = 0.005$.

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