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SOME CLASSES OF "TRANSMUTED ADAPTIVE FUNCTIONS". APPLICATIONS

Nikolay Kyurkchiev[1], Nikolay Pavlov[2],
Anton Iliev[3] and Asen Rahnev[4]

[1,2,3,4]Faculty of Mathematics and Informatics
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA

ABSTRACT: In this article we will consider some methodological aspects related to the possibility of generating new classes of adaptive functions based on the generalized transmuted family proposed by Shaw and Buckley [1].

Some applications are also given.

Numerical examples, illustrating our results using *CAS MATHEMATICA* are given.

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Key Words: quadratic rank transformation map, generalized SIN–G "adaptive function", Hausdorff distance, modified families of functions with "polynomial variable transfer", antenna diagrams, simulation–correction function

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1. INTRODUCTION

The quadratic rank transformation map (QRTM) proposed by Shaw and Buckley [1] having the form

$$M(t) = (1+\lambda)F(t) - \lambda F^2(t) \tag{1}$$

where $|\lambda| < 1$.

Let us consider the generalized SIN–G family [3]

$$F(t) = \sin\left(\frac{\pi}{2}G(t)\right); \;\; t \in R \tag{2}$$

where $G(t)$ denotes baseline cdf $G(t) = 1 - e^{-bt}$.

From (1) we have

$$\begin{aligned} M_1(t) &= (1+\lambda)\sin\left(\tfrac{\pi}{2}(1-e^{-bt})\right) - \lambda \sin^2\left(\tfrac{\pi}{2}(1-e^{-bt})\right) \\ &= \sin\left(\tfrac{\pi}{2}(1-e^{-bt})\right)\left(1+\lambda-\lambda\sin\left(\tfrac{\pi}{2}(1-e^{-bt})\right)\right). \end{aligned} \tag{3}$$

Consider the following new class of SIN–G family proposed by Mahmood and Chesneau [2]

$$F(t) = \sin\left(\frac{\pi}{4}G(t)(1+G(t))\right); \; t \in R. \tag{4}$$

Consider the new class with $G(t) = 1 - e^{-bt}$, i.e.:

$$M_4(t) = \sin\left(\frac{\pi}{4}(1-e^{-bt})(2-e^{-bt})\right). \tag{5}$$

The detailed study of the new families (3) and (5) in terms of properties, saturation (in Hausdorff sense [13]), applications etc. is the subject of the proposed article.

For other results, see [4]–[12].

2. MAIN RESULTS

2.1. A LOOK AT THE FAMILY (3)

Let $t = r\cos\theta + c$, where θ is the azimuthal angle. Some classes of antenna diagrams can be described as:

$$H(\theta) = H_1(\theta)H_a(\theta), \tag{6}$$

where $H_1(\theta)$ is the element factor, and $H_a(\theta)$ is the array factor.

Obviously, the new transmuted adaptive model (3) is of the type (6), which makes it very attractive when conducting simulations in the field of antenna–feeder analysis.

2.2. NUMERICAL EXPERIMENTS USING $M_1(T)$

1. For the "saturation" - d to the horizontal asymptote in the Hausdorff sense by means of function $M_1(t)$ we have:

$$M_1(d) - 1 + d = 0. \tag{7}$$

The family $M_1(t)$ for

a) $b = 10$; $\lambda = 0.05$; Hausdorff distance $d = 0.115255$;
b) $b = 30$; $\lambda = 0.95$; Hausdorff distance $d = 0.0332948$

is plotted on Figure 1.

From these and additional experiments it can be concluded that the model $M_1(t)$ is very sensitive to the choice of the parameter λ, which makes it attractive to users.

The corresponding adaptive function with "polynomial variable transfer" is of the type:

$$\begin{aligned} M_1^*(t) &= (1+\lambda)\sin\left(\tfrac{\pi}{2}(1-e^{-b|f(t)|})\right) - \lambda\sin^2\left(\tfrac{\pi}{2}(1-e^{-b|f(t)|})\right), \\ f(t) &= \sum_{i=0}^{n} a_i t^i, \quad a_0 = 0. \end{aligned} \tag{8}$$

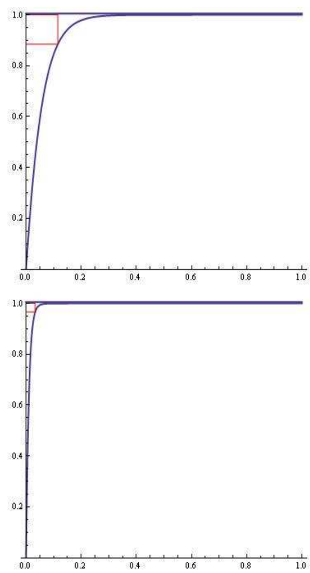

Figure 1: The family $M_1(t)$ for a) $b = 10$; $\lambda = 0.05$; Hausdorff distance $d = 0.115255$; b) $b = 30$; $\lambda = 0.95$; Hausdorff distance $d = 0.0332948$.

2. Example 1. Let

$$f(t) = t(1-t)(2-t)(3-t)(4-t)(5-t)(6-t)(7-t).$$

The adaptive function with polynomial variable transfer $M_1^*(t)$ and $M_1^*(\theta)$; $t = r\cos\theta + c$ for $b = 0.0006$; $\lambda = 0.1$; $r = -3.2$; $c = -0.4$ are plotted on Figure 2.

In many cases, the antenna diagram is subject to a number of restrictions related to the intensity of radiation in fixed sectors.

The following example is illustrative.

Example 2. Let the function $f(t)$ (as a simulation–correction function) be selected in the way visualized in Figure 3 a).

The function $M_1^*(\theta)$; $t = r\cos\theta + c$ for $b = 0.05$; $\lambda = 0.9$; $r = -4.9$; $c = -1.95$ is plotted on Fig. 3 b).

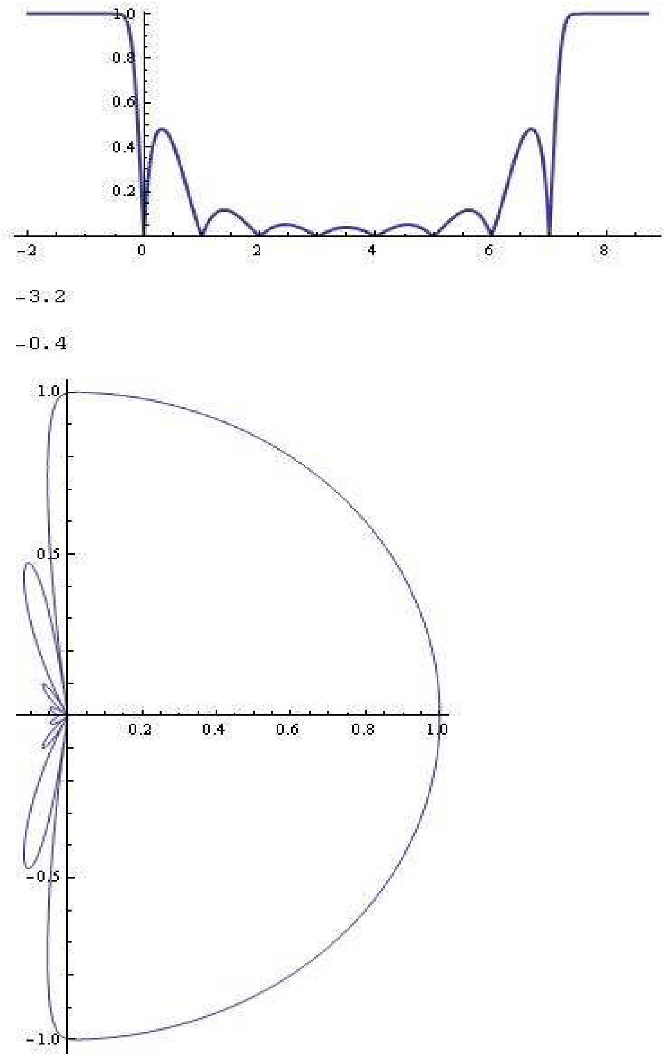

Figure 2: a) the family $M_1^*(t)$; b) $M_1^*(\theta)$. Example 1.

2.3. A LOOK AT THE NEW SIN–G "TRANSMUTED FAMILY" BY MAHMOOD AND CHESNEAU [2]

We will note that the constructive approach proposed by authors in [2] is very close to the idea of generating "transmuted" cumulative distribution functions.

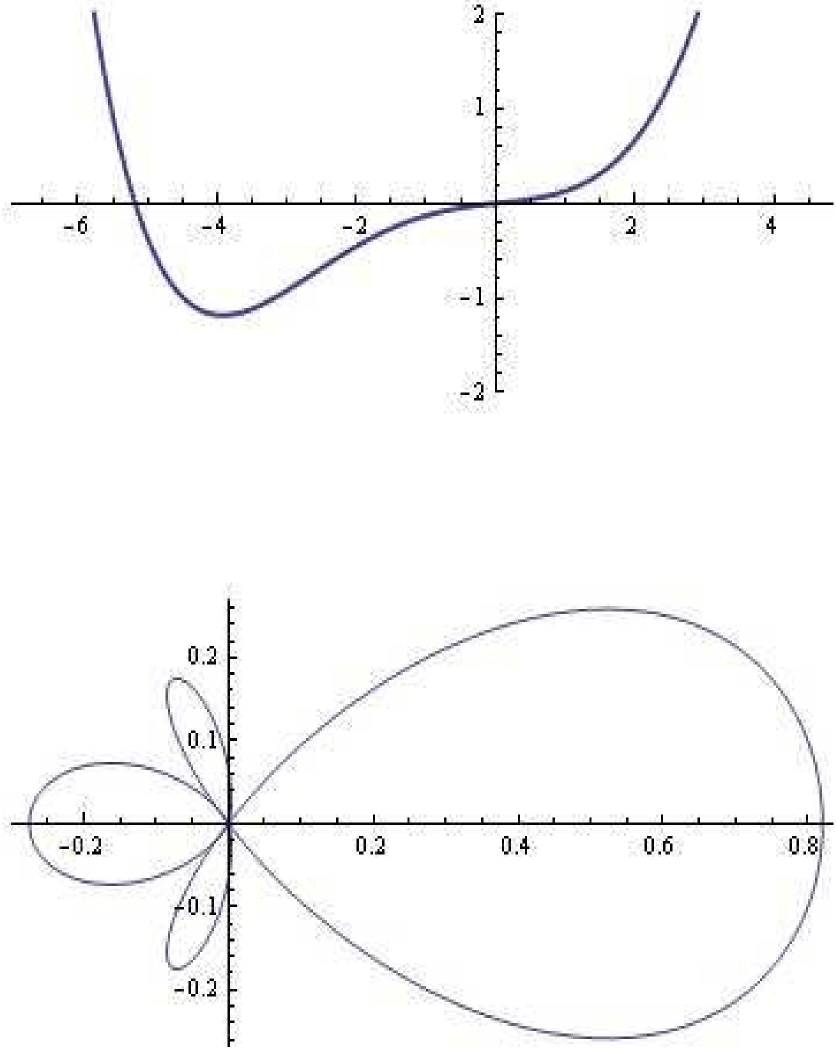

Figure 3: a) the simulation–correction function $f(t)$; b) The family $M_1^*(\theta)$. Example 2.

2.3.1. HAUSDORFF APPROXIMATION OF THE HEAVISIDE STEP FUNCTION BY $M_4(T)$

Let t_0 is the solution of the nonlinear equation

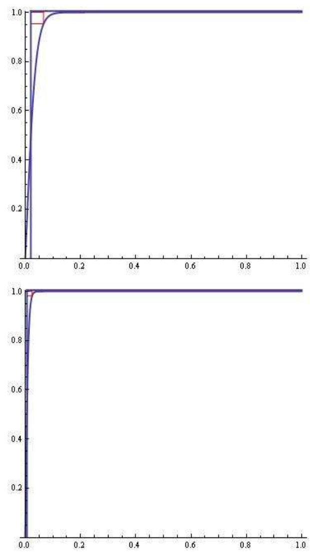

Figure 4: The function M_4 for: a) $b = 30$; $t_0 = 0.0203811$; Hausdorff distance $d = 0.0462067$; b) $b = 100$; $t_0 = 0.00611433$; Hausdorff distance $d = 0.0186099$.

$$M_4(t_0) - \frac{1}{2} = 0$$

or

$$(e^{-bt_0})^2 - 3e^{-bt_0} + 2 - \frac{4}{\pi}\arcsin(0.5) = 0.$$

Evidently, for the appropriate "median level" we have

$$t_0 = -\frac{1}{b}\ln 0.542573.$$

For the Hausdorff approximation - d of the Heaviside step function $h_{t_0}(t)$ by $M_4(t)$ we find

$$M_4(t_0 + d) - 1 + d = 0. \qquad (9)$$

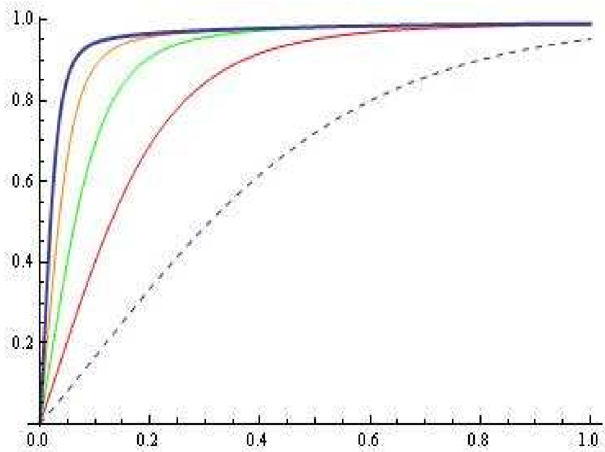

Figure 5: The graphics of recurrence generated adaptive functions: γ_0 (dashed), γ_1 (red), γ_2 (green), γ_3 (orange) and γ_4 (thick) for fixed $b = 1.7$.

2.3.2. NUMERICAL EXPERIMENTS

The family $M_4(t)$ for

a) $b = 30$; $t_0 = 0.0203811$; Hausdorff distance $d = 0.0462067$ (d is computed from nonlinear equation (9));

b) $b = 100$; $t_0 = 0.00611433$; Hausdorff distance $d = 0.0186099$

is plotted on Figure 4.

2.3.3. A FAMILY OF RECURRENCE GENERATED FUNCTIONS BASED ON (5)

We construct a family of recurrence generated functions by

$$\delta_{i+1}(t) = \sin\left(\tfrac{\pi}{4}\left(1 - e^{-b(t+\delta_i(t))}\right)\left(2 - e^{-b(t+\delta_i(t))}\right)\right) \qquad (10)$$

$$i = 0, 1, 2, \ldots$$

with

$$\delta_0(t) = M_4(t);\ \ \delta_0(0) = 0.$$

The recurrence generated: $\delta_0(t), \delta_1(t), \delta_2(t), \delta_3(t)$ and $\delta_4(t)$ from family (10) for fixed $b = 1.98$ are visualized on Figure 5.

2.3.4. A NEW CLASS OF "ADAPTIVE FUNCTION" WITH "POLYNOMIAL VARIABLE TRANSFER". APPLICATIONS

Formally, we define the following "adaptive function" with "polynomial variable transfer":

$$\begin{aligned} &M_4^*(t) = \sin\left(\tfrac{\pi}{4}\left(1 - e^{-b|f(t)|}\right)\left(2 - e^{-b|f(t)|}\right)\right), \\ &f(t) = \sum_{i=0}^{n} a_i t^i, \quad a_0 = 0. \end{aligned} \tag{11}$$

Consider the adaptive function

$$M_4^*(t) = \sin\left(\frac{\pi}{4}\left(1 - e^{-b|t(1-t)(2-t)(3-t)(4-t)(5-t)|}\right) \times \left(2 - e^{-b|t(1-t)(2-t)(3-t)(4-t)(5-t)|}\right)\right).$$

For $b = 0.0028$ the function $M_4^*(t)$ is depicted on Figure 6.

Example. Let $t = r\cos\theta + c$, where θ is the azimuthal angle.

Then, typical emitting charts using $M_4^*(\theta)$ for

$n = 5,\ b = 0.1,\ a_0 = 0,\ a_1 = -1.9,\ a_2 = -0.3,\ a_3 = 3.7,\ a_4 = -0.1,\ a_5 = -0.85,\ r = -1.2,\ c = -0.22$

is plotted on Figure 7.

Remark. The comparisons with other models show that the proposed new adaptive function $M_4^*(t)$ (see, also $M_4^*(\theta)$) achieves a lower level of possible noise in the antenna.

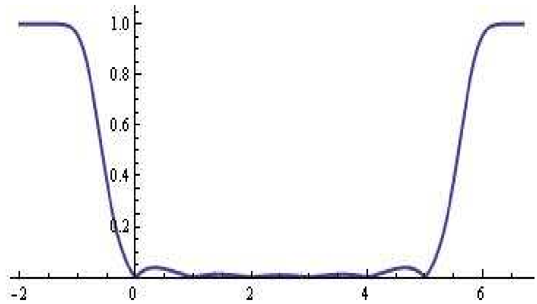

Figure 6: The adaptive function $M_4^*(t)$ for $b = 0.0028$.

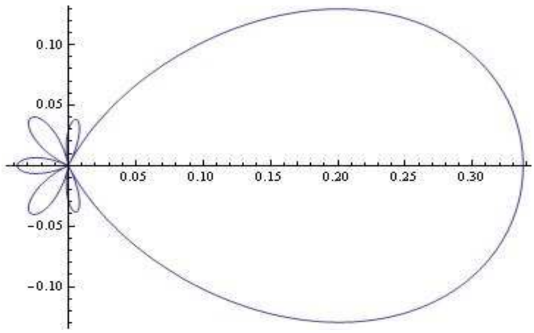

Figure 7: A typical radiation pattern using $M_4^*(\theta)$ for $n = 5$, $b = 0.1$, $a_0 = 0$, $a_1 = -1.9$, $a_2 = -0.3$, $a_3 = 3.7$, $a_4 = -0.1$, $a_5 = -0.85$, $r = -1.2$, $c = -0.22$ in $(-\pi, \pi)$).

This makes the new model (11) interesting for researchers working in this scientific field.

For other models, see [14]–[23].

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