

**ON THE POSSIBILITY OF GOOD SUPPLY OF SOME  
"SPECIFIC DATA" WITH ONE MODIFIED  $\sinh$   
INVERTED EXPONENTIAL MODEL**

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**ABSTRACT:** In this paper we study some properties of the  $\sinh$  inverted exponential cumulative model (SIE) [1]. More precisely - the "saturation" in the Hausdorff sense. A good supply of some specific data (for example, "Neck cumulative cancer data") by a modified model with "polynomial variable transfer" (SIEPVT) are also given. Numerical examples are presented using *CAS MATHEMATICA*.

**AMS Subject Classification:**  $\sinh$  inverted exponential cumulative model (SIE), "Saturation" in the Hausdorff sense, modified model with "polynomial variable transfer" (SIEPVT)

**Key Words:** 41A46

**Received:** July 7, 2020;      **Accepted:** November 9, 2020;

**Published:** November 14, 2020      **doi:** 10.12732/caa.v24i1.5

Dynamic Publishers, Inc., Acad. Publishers, Ltd.      <http://www.acadsol.eu/caa>

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## 1. INTRODUCTION

**Definition 1.** In [1] the authors proposed the following new  $\sinh$  inverted exponential distribution (SIE) with cumulative distribution function:

$$F(t; a, k) = \frac{2e^a}{(e^a - 1)^2} \left( \cosh \left( ae^{-\frac{k}{t}} \right) - 1 \right) \quad (1)$$

for  $t > 0$ ,  $a > 0$ ,  $k > 0$ .

**Definition 2.** The Hausdorff distance [5] (the H-distance)  $\rho(f, g)$  between two interval functions  $f, g$  on  $\Omega \subseteq \mathbb{R}$ , is the distance between their completed graphs  $F(f)$  and  $F(g)$  considered as closed subsets of  $\Omega \times \mathbb{R}$ . More precisely,

$$\rho(f, g) = \max \left\{ \sup_{A \in F(f)} \inf_{B \in F(g)} \|A - B\|, \sup_{B \in F(g)} \inf_{A \in F(f)} \|A - B\| \right\},$$

wherein  $\|\cdot\|$  is any norm in  $\mathbb{R}^2$ , e. g. the maximum norm  $\|(t, x)\| = \max\{|t|, |x|\}$ ; hence the distance between the points  $A = (t_A, x_A)$ ,  $B = (t_B, x_B)$  in  $\mathbb{R}^2$  is  $\|A - B\| = \max(|t_A - t_B|, |x_A - x_B|)$ .

**Definition 3.** We define the following new model with "polynomial variable transfer" (SIEPVT):

$$M(t) = A \frac{2e^a}{(e^a - 1)^2} \left( \cosh \left( ae^{-\frac{k}{f(t)}} \right) - 1 \right), \quad (2)$$

$$f(t) = \sum_{i=0}^n a_i t^i; \quad a_0 = 0.$$

In this note we study some properties of the model (1). The possibility of good supply of some "specific data" by the new modified  $\sinh$  inverse exponential model (SIEPVT) (2) is also discussed.

## 2. MAIN RESULTS. NUMERICAL EXAMPLES

When studying the intrinsic properties of the  $F(t; a, k)$ , it is also appropriate to study the "saturation" to the horizontal asymptote.

For the "saturation" -  $d$  in the Hausdorff sense [5] we have the following nonlinear equation

$$F(d; a, k) = 1 - d. \quad (3)$$

The model (1) for fixed:

- 1)  $k = 0.01$ ,  $a = 0.11$  and H-distance  $d = 0.136453$ ;
- 2)  $k = 0.0005$ ,  $a = 0.1$  and H-distance  $d = 0.0975272$

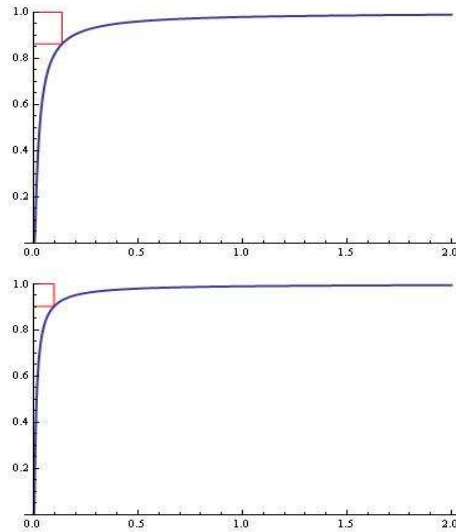


Figure 1: The model (1) for 1)  $k = 0.01$ ,  $a = 0.11$  and H-distance  $d = 0.136453$ ; 2)  $k = 0.0005$ ,  $a = 0.1$  and H-distance  $d = 0.0975272$ .

is plotted on Fig. 1.

Let  $A = 1$ . Similarly for the "saturation" -  $d_1$  by model (2) we have

$$M(d_1) = 1 - d_1. \tag{4}$$

The model (2) for fixed

1)  $k = 0.01$ ,  $a = 0.11$ ;  $a_0 = 0$ ,  $a_1 = 20$ ,  $a_2 = -8$ ,  $a_3 = 0.9$  and H-distance  $d_1 = 0.0851109$ ;

2)  $k = 0.0005$ ,  $a = 0.1$ ;  $a_0 = 0$ ,  $a_1 = 20$ ,  $a_2 = -8$ ,  $a_3 = 0.9$  and H-distance  $d_1 = 0.0652741$

is plotted on Fig. 2.

A comparison of Figures 1 and 2 shows that the saturation  $d_1$  is better than  $d$ .

This gives us reason to use model (2) in approximating some specific data.

**Example 1.** The following data is used in the modelling process [6] (see, Fig. 3)

For the actual values in the specified period the model  $M(t)$  for

$a = 1.38$ ,  $k = 1.163$ ,  $n = 4$ ,  $A = 46.8944$ ,  $a_0 = 0$ ,  $a_1 = 6.24175$ ,  $a_2 = -10.3027$ ,

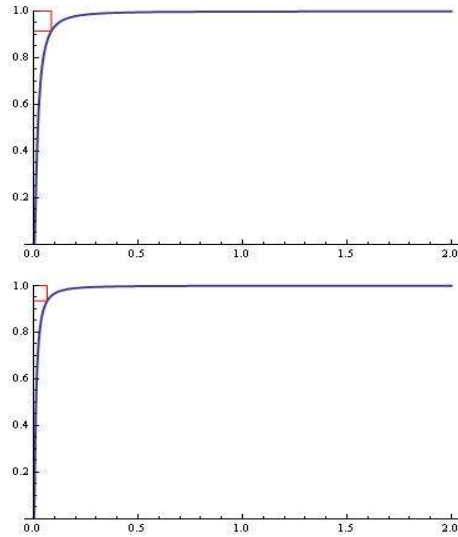


Figure 2: The model (2) for 1)  $k = 0.01$ ,  $a = 0.11$ ;  $a_0 = 0$ ,  $a_1 = 20$ ,  $a_2 = -8$ ,  $a_3 = 0.9$  and H-distance  $d_1 = 0.0851109$ ; 2)  $k = 0.0005$ ,  $a = 0.1$ ;  $a_0 = 0$ ,  $a_1 = 20$ ,  $a_2 = -8$ ,  $a_3 = 0.9$  and H-distance  $d_1 = 0.0652741$ .

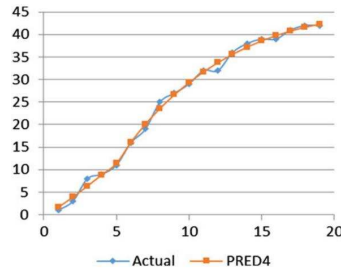


Figure 3: Goodness of fit curve for release 4 [6].

$$a_3 = 12.4721, a_4 = -2.82035$$

is depicted on Fig. 4.

(We have adopted a scale on the horizontal axis: 0.1 division corresponds to 1 time interval).

**Example 2.** The applicability of the model (1) is proved in simulation study to "neck cumulative cancer data" in [1] (see, Fig. 5)

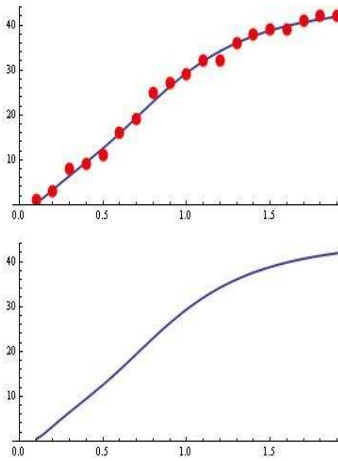


Figure 4: a) The "actual data"; b) The fitted model  $M(t)$  ( $a = 1.38$ ,  $k = 1.163$ ,  $n = 4$ ,  $A = 46.8944$ ,  $a_0 = 0$ ,  $a_1 = 6.24175$ ,  $a_2 = -10.3027$ ,  $a_3 = 12.4721$ ,  $a_4 = -2.82035$ ) (Example 1).

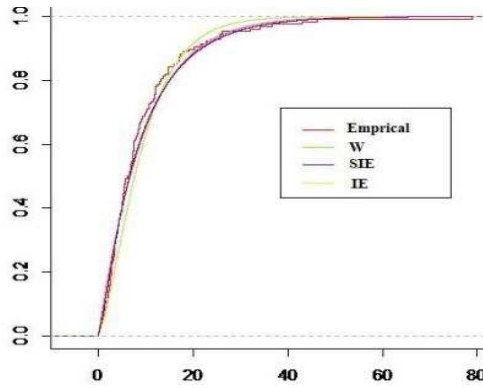


Figure 5: The Weibull, SIE and inverse exponential models for the "neck cumulative cancer data" (see, [1]).

For the actual data in the specified period the our new model  $M(t)$  for

$$a = 0.75, k = 0.4, n = 4, A = 0.978191, a_0 = 0, a_1 = 1.9648, a_2 = -2.11186,$$

$$a_3 = 2.67002, a_4 = -0.293813$$

is depicted on Fig. 6.

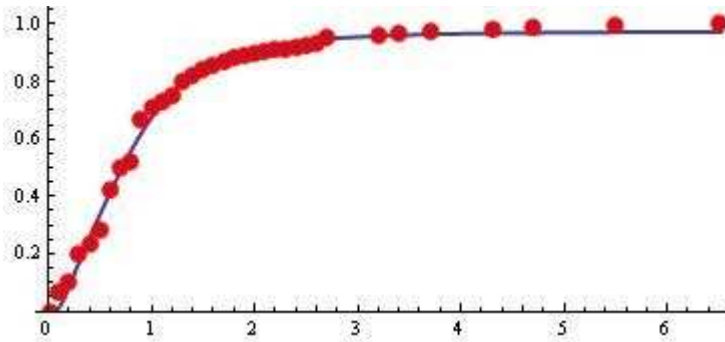


Figure 6: The fitted model  $M(t)$  ( $a = 0.75$ ,  $k = 0.4$ ,  $n = 4$ ,  $A = 0.978191$ ,  $a_0 = 0$ ,  $a_1 = 1.9648$ ,  $a_2 = -2.11186$ ,  $a_3 = 2.67002$ ,  $a_4 = -0.293813$ ) for the "neck cumulative cancer data" (Example 2).

(We have adopted a scale on the horizontal axis: 0.1 division corresponds to 1 time interval).

**Example 3.** Approximation of the data "Influenza A(H1N1) (also known as Swine influenza)" [3].

In April 2009 scientific laboratories discover an influenza virus known as H1N1. This virus seems like a combination of viruses from birds, humans and pigs.

Later, in 2010 The World Health Organization (WHO) reported the infection caused by H1N1 as a global pandemic.

In August 2010 the WHO reported the finishing of pandemic. Since that scientists changed the name of infection and H1N1 virus is now H1N1v.

For the cumulative data from Fig. 7 the fitted model  $M(t)$  for  $a = 1.5$ ,  $k = 1.174$ ,  $n = 6$ ,  $A = 880$ ,  $a_0 = 0$ ,  $a_1 = -9.64272$ ,  $a_2 = 139.487$ ,  $a_3 = -502.948$ ,  $a_4 = 864.814$ ,  $a_5 = -701.235$ ,  $a_6 = 216.54$  (Example 3) is presented on Fig. 8.

**Example 4.** The Fig. 9 show the time-dependent behavior of the estimated mean value functions with the effect of the change-point on the software reliability growth process [4].

For the cumulative data from Fig. 9 the fitted model  $M(t)$  for  $a = 1.48$ ,  $k = 1.19$ ,  $n = 6$ ,  $A = 40$ ,  $a_0 = 0$ ,  $a_1 = -2.39748$ ,  $a_2 = 45.2554$ ,  $a_3 = -141.64$ ,  $a_4 = 186.856$ ,  $a_5 = -109.239$ ,  $a_6 = 23.5688$  (Example 4) is presented on Fig. 10.

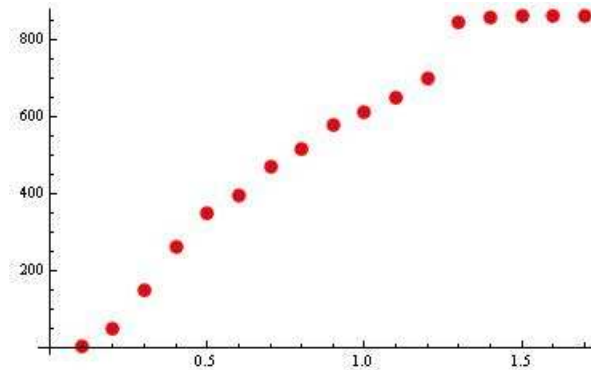


Figure 7: Influenza A(H1N1) cases [3].

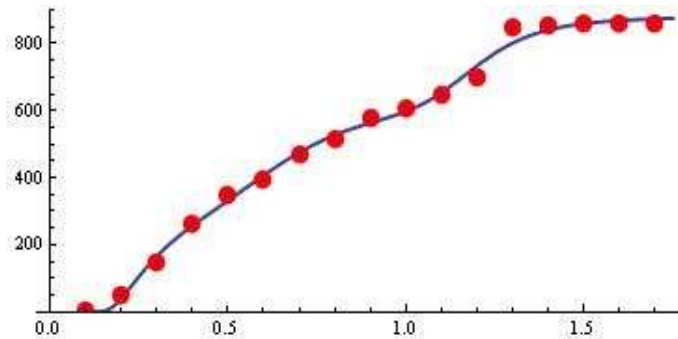


Figure 8: The fitted model  $M(t)$  for  $a = 1.5$ ,  $k = 1.174$ ,  $n = 6$ ,  $A = 880$ ,  $a_0 = 0$ ,  $a_1 = -9.64272$ ,  $a_2 = 139.487$ ,  $a_3 = -502.948$ ,  $a_4 = 864.814$ ,  $a_5 = -701.235$ ,  $a_6 = 216.54$  (Example 3).

### 3. CONCLUDING REMARKS

The use of the new model with many free parameters  $a_i$ ;  $i = 1, 2, \dots$ , makes it attractive for analysis and approximation of specific data from Population Dynamics, Biostatistics, Debugging and Test Theory and Computer viruses propagation.

From the analysis of the conducted experiment it is clear that the proposed new model behaves extremely adequately in approximating data in the field of Growth theory and Debugging and Test Theory.

The only drawback is the fact that the model is very sensitive to the

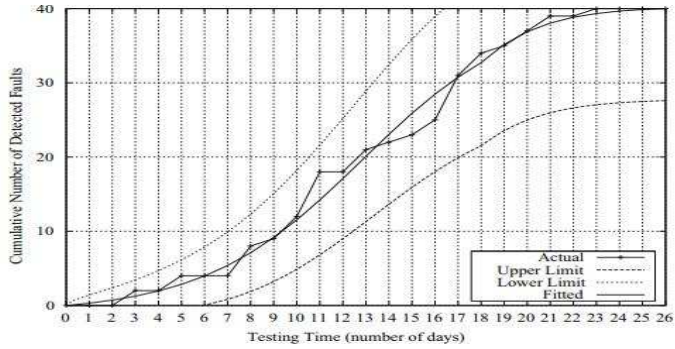


Figure 9: The real data - Example 4 (see, [4]).

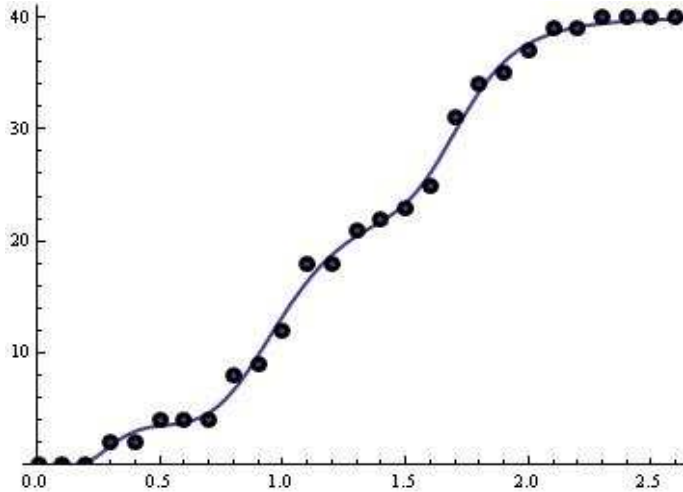


Figure 10: The fitted model  $M(t)$  for  $a = 1.48$ ,  $k = 1.19$ ,  $n = 6$ ,  $A = 40$ ,  $a_0 = 0$ ,  $a_1 = -2.39748$ ,  $a_2 = 45.2554$ ,  $a_3 = -141.64$ ,  $a_4 = 186.856$ ,  $a_5 = -109.239$ ,  $a_6 = 23.5688$  (Example 4).

number and distribution of zeros of polynomial  $f$  [2].

We recommend the use of this model at a degree of polynomial  $n \leq 10$ .

In many cases, due to the large number of free parameters -  $a_i$ , standard program operators, implemented in different programming environments and designed to minimize heavy functionality (of many variables) - do not work or do not give satisfactory results.

This necessitated, when conducting the numerical examples contained in



this article, to write a specialized program module, which is in a sense - overriding (for example, for the software environment *CAS Mathematica*).

For other results, see [7]–[38].

## ACKNOWLEDGMENTS

This work has been accomplished with the financial support by the Grant No BG05M2OP001-1.001-0003, financed by the Science and Education for Smart Growth Operational Program (2014-2020) and co-financed by the European Union through the European structural and Investment funds.

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