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COMMENTS ON A NEW EXTENDED LOMAX–INVERSE–LINDLEY FAMILY OF C.D.F.

ANNA MALINOVA[1], OLGA RAHNEVA[2], TODORKA TERZIEVA[3], AND EVGENIA ANGELOVA[4]

[1,3,4]Faculty of Mathematics and Informatics
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA

[2]Faculty of Economy and Social Sciences
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA

ABSTRACT: In [1] the authors propose the following new extended Lomax–Inverse–Lindley family of c.d.f.:

$$M(t) = 1 - \beta^{\alpha}\left(\beta - Log[1 - \left(1 + \frac{\theta}{(\theta+1)t}\right)e^{-\frac{\theta}{t}}]\right)^{-\alpha},$$

where $(\alpha, \beta, \theta) \in R^{+}$.

Some properties and applications to the lifetime data are given.

Also of interest to the specialists is the task of approximating the Heaviside function

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases}$$

where t_0 is the median with the new cumulative function in the Hausdorff sense.

We define a new family of recurrence generated cdf of the transmuted extended Lomax–Inverse–Lindley (TELIL) distribution

$$M_{i+1}(t) = M_i(t)(\lambda_{i+1} + 1 - \lambda_{i+1}M_i(t)),$$

$$i = 0, 1, 2, \ldots, \quad \lambda_i \in [0, 1); \; M_0(t) = M(t).$$

Numerical examples, illustrating our results are presented using programming environment *CAS Mathematica*.

AMS Subject Classification: 41A46

Key Words: extended Lomax–Inverse–Lindley (ELIL) distribution, family of recurrence generated cdf of the transmuted extended Lomax–Inverse–Lindley (TELIL) distribution, Heaviside step–function $h_{t_0}(t)$, Hausdorff distance

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1. INTRODUCTION AND PRELIMINARIES

Definition 1. *In [1] the authors consider the following new extended Lomax–Inverse–Lindley family of c.d.f.:*

$$M(t) = 1 - \beta^{\alpha} \left(\beta - Log[1 - \left(1 + \frac{\theta}{(\theta+1)t} \right) e^{-\frac{\theta}{t}}] \right)^{-\alpha}, \tag{1}$$

where $(\alpha, \beta, \theta) \in R^{+}$, $t > 0$.

Definition 2. *The shifted Heaviside step function is defined by*

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases} \tag{2}$$

Definition 3. *[14] The Hausdorff distance (the H–distance)* $\rho(f, g)$ *between two interval functions* f, g *on* $\Omega \subseteq \mathbb{R}$, *is the distance between their completed graphs* $F(f)$ *and* $F(g)$ *considered as closed subsets of* $\Omega \times \mathbb{R}$. *More precisely,*

$$\rho(f, g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B|| \},$$

wherein $||.||$ *is any norm in* $\mathbb{R}^2$*, e. g. the maximum norm* $||(t,x)|| = \max\{|t|,|x|\}$*; hence the distance between the points* $A = (t_A, x_A)$*,* $B = (t_B, x_B)$ *in* $\mathbb{R}^2$ *is* $||A - B|| = max(|t_A - t_B|, |x_A - x_B|)$.

Definition 4. *We define a new family of recurrence generated cdf of the transmuted extended Lomax–Inverse–Lindley (TELIL) distribution:*

$$\begin{aligned} &M_{i+1}(t) = M_i(t)(\lambda_{i+1} + 1 - \lambda_{i+1} M_i(t)), \\ &i = 0, 1, 2, \dots, \quad \lambda_i \in [0,1); \;\; M_0(t) = M(t). \end{aligned} \tag{3}$$

For some generalized family of distributions, see [2]–[13].

In this note we study the Hausdorff approximation of the Heaviside function $h_{t_0}(t)$ by the family $M_i(t)$.

2. MAIN RESULTS

2.1. A NOTE ON THE NEW EXTENDED LOMAX–INVERSE–LINDLEY FAMILY (ELIL) OF CDF (1)

The investigation of the characteristic "supersaturation" of the model (1) to the horizontal asymptote is important.

Sensitive analysis for the "saturation in the Hausdorff sense".

Let t_0 is the value for which $M(t_0) = \frac{1}{2}$.

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the (cdf) $M(t)$ (1) satisfies the relation

$$M(t_0 + d) = 1 - d. \tag{4}$$

For given α, β, θ and t_0, the nonlinear equation (4) has unique positive root – d.

The model (1) for $\theta = 3$, $\alpha = 10$, $\beta = 0.5$ and $t_0 = 0.741859$ is visualized on Fig. 1.

From the nonlinear equation (4) we have: $d = 0.219605$.

The model (1) for $\theta = 3$, $\alpha = 10$, $\beta = 0.05$ and $t_0 = 0.454065$ is visualized on Fig. 2.

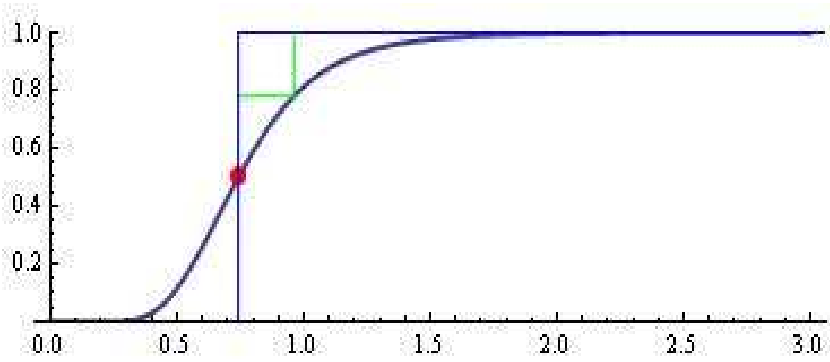

Figure 1: The model (1) for $\theta = 3$, $\alpha = 10$, $\beta = 0.5$ and $t_0 = 0.741859$ H–distance $d = 0.219605$.

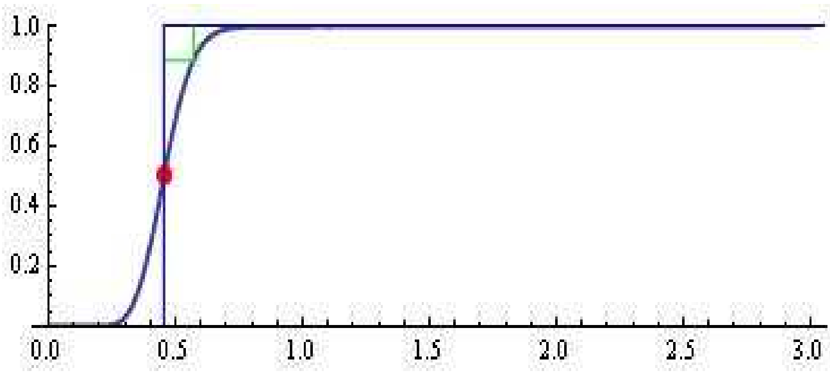

Figure 2: The model (1) for $\theta = 3$, $\alpha = 10$, $\beta = 0.05$ and $t_0 = 0.454065$; H–distance $d = 0.115652$.

From the nonlinear equation (4) we have: $d = 0.115652$.

The model (1) for $\theta = 3$, $\alpha = 10$, $\beta = 0.005$ and $t_0 = 0.328925$ is visualized on Fig. 3.

From the nonlinear equation (4) we have: $d = 0.0689189$.

Example. Storm worm one of the most biggest cyber threats of 2008.

We analyze the following data [15]

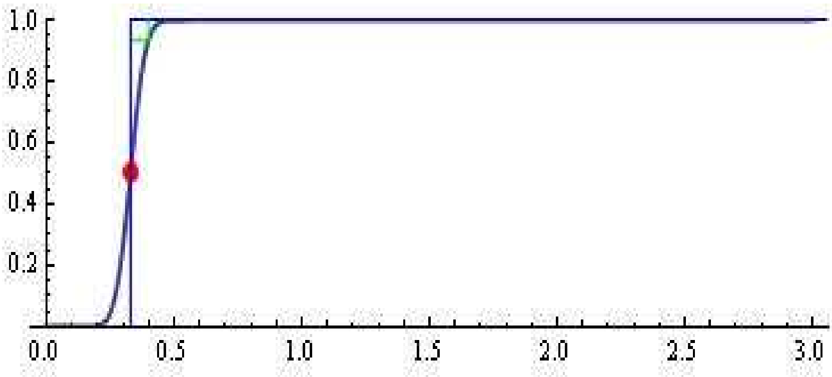

Figure 3: The model (1) for $\theta = 3$, $\alpha = 10$, $\beta = 0.005$ and $t_0 = 0.328925$; H–distance $d = 0.0689189$.

$$data\_Storm\_IDs := \{\{1, 0.843\}, \{4, 0.926\}, \{5, 0.954\}, \{6, 0.967\}, \{7, 0.976\}, \{8, 0.981\}, \{9, 0.985\}, \{10, 0.991\}, \{22, 0.995\}, \{38, 0.997\}, \{51, 0.998\}, \{64, 0.9985\}, \{74, 0.999\}, \{83, 1\}, \{100, 1\}, \{367, 1\}\}$$

The cdf $M(t)$ for $\alpha = 3.211022905$, $\theta = 1.807332018$ and $\beta = 0.9$ is visualized on Fig. 4.

For the

$$error = \sum_i \left(\frac{M(t_i) - y_i}{y_i} \right)^2$$

we have $error = 0.000818157$.

The ”saturation” in the Hausdorff sense is visualized on Fig. 5.

2.2. A NOTE ON THE NEW FAMILY OF RECURRENCE GENERATED CDF OF THE TRANSMUTED EXTENDED LOMAX–INVERSE–LINDLEY (TELIL) DISTRIBUTION (3)

Sensitive analysis for the ”saturation in the Hausdorff sense”.

For $i = 0$ from (3) we have:

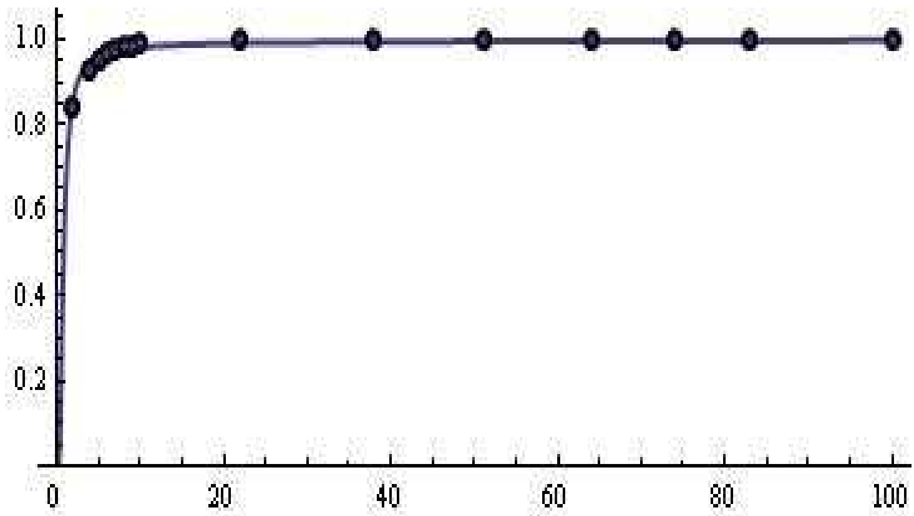

Figure 4: The fitted model $M(t)$ with $error = 0.000818157$.

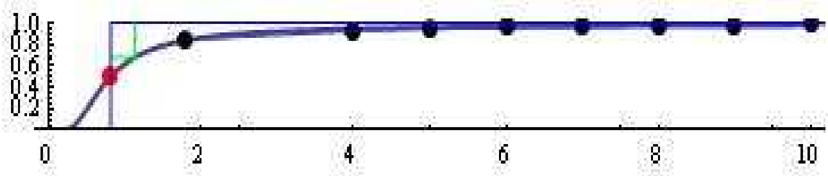

Figure 5: The "saturation" in the Hausdorff sense.

$$M_1(t) = M(t)(\lambda_1 + 1 - \lambda_1 M(t)) \tag{5}$$

Let t_1 is the value for which $M_1(t_1) = \frac{1}{2}$.

The one–sided Hausdorff distance d_1 between the function $h_{t_1}(t)$ and the (cdf) $M_1(t)$ (5) satisfies the relation

$$M_1(t_1 + d_1) = 1 - d_1. \tag{6}$$

For example, for fixed $\theta = 3$, $\alpha = 10$, $\beta = 0.5$, $\lambda_1 = 0.6$ and $t_1 = 0.660027$ for the one–sided H–distance we find $d_1 = 0.19802$ (see, Fig. 6).

The recurrence generated cumulative distribution functions: $M(t)$, $M_1(t)$, $M_2(t)$ and $M_3(t)$ are visualized on Fig. 7.

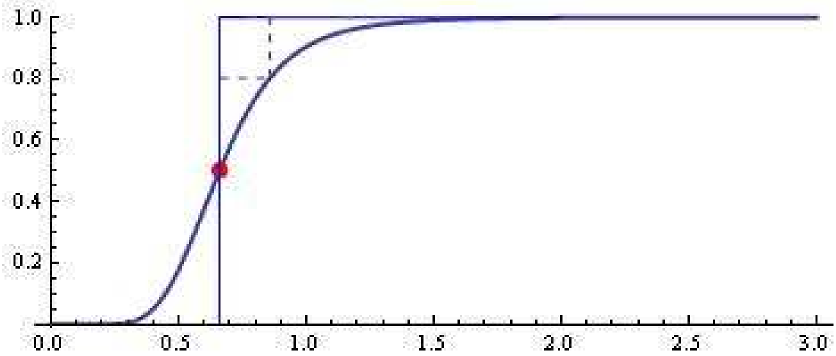

Figure 6: The model (3) for $\theta = 3$, $\alpha = 10$, $\beta = 0.5$, $\lambda_1 = 0.6$, $t_1 = 0.660027$; H–distance $d_1 = 0.19802$.

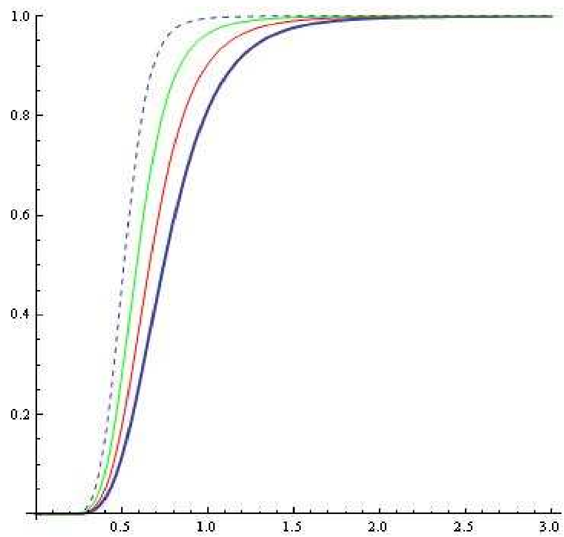

Figure 7: Comparison between $M(t)$–thick, $M_1(t)$–red, $M_2(t)$–green and $M_3(t)$–dashed for fixed $\theta = 3$, $\alpha = 10$, $\beta = 0.5$ and $\lambda_1 = 0.6$, $\lambda_2 = 0.7$, $\lambda_3 = 0.9$.

Example. Application of the new cumulative sigmoid for analysis of the "cancer data" [36]–[37]

We will illustrate the advances of the new family for approximation and modelling of "cancer data" (for some details see, [36]–[37]).

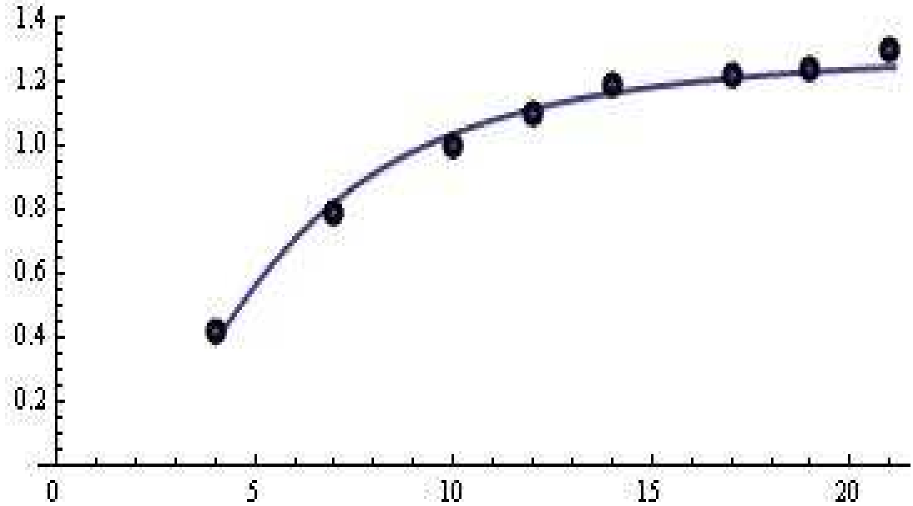

Figure 8: The model $M_1(t)$ based on the "cancer data".

days	4	7	10	12	14	17	19	21
$R(t)$	0.415	0.794	1.001	1.102	1.192	1.22	1.241	1.3

Table 1: The "cancer data" [36]–[37]

The model $M_1(t)$ based on the data from Table 1 for the estimated parameters:

$$\omega = 1.3;\ \lambda_1 = 0.999;\ \beta = 15.5;\ \alpha = 22.8322;\ \theta = 9.66431$$

is plotted on Fig. 8.

Obviously, the new model (5) can also be used for approximating of some "specific data".

It can be seen that the "supersaturation" by the (cdf) $M_i(t)$ is faster.

Evidently, $\{d_i\}_1^\infty \to 0$.

For other approximation and modelling results, see [16]–[35].

We hope that the results will be useful for specialists in this scientific area.

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