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PRODUCTION PLANNING AND ENGINEERING PROCESS IMPROVEMENT

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ABSTRACT: The authors present an optimal control model to allocated equipment capacity between production that generates immediate revenue and engineering process improvement activity that results in increased future output. The benefit of engineering activity is modeled as a concave function of the total number of engineering lots processed to date, while the production facility is represented by a nonlinear clearing function capturing the nonlinear relationship between resource utilization and cycle time. We analyze the model to develop structural results and illustrates its behavior with numerical experiments.

Key Words: optimal control model, engineering process, structural results

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1. INTRODUCTION

Due to the high sale prices that can be obtained early in a product life cycle, the ability to rapidly increase the production value of good devices is often a critical advantage for semiconductor firms. Management must allocate equipment capacity between production lots, which will lead to a saleable product and generate revenue and engineering activity that will not yield revenue immediately but will increase output of good product in the future. The benefits of engineering work are usually seen after a time lag that elapses while the results are analyzed, problems diagnosed, and remedial actions implemented

We start with discrete model ([13]) followed by a continuous time optimal control version([5]). One can consider a series of models of improvement activities in production systems where improvement activities take place on the production equipment, reducing its capacity to produce saleable output. We begin with a simple single item model where the amount of improvement activity is a concave nondecreasing function of the cumulative improvement activity taken at time t. In future work we consider models with switching times where the dynamics of the problem is updated considering the status of work in process inventory (WIP), finished good inventory, and number of production lots are updated.

The organization of the paper is as follows. We first present preliminaries ([13]), then the problem statement. Then, we present control problem formulation of the problem, and finally present computational results.

2. PRELIMINARIES

We distinguish production from engineering activity, which is represented by engineering lots, and the learning that increases capacity results from the cumulative number of engineering lots processed to date. In contrast to representing the effects of learning as a decrease in unit resource requirements, we represent their effects as an increase in the output of good product in a planning period. We postulate a capacity improvement function by ϕ.

$$\phi(\sum_{\tau=0}^{t-d} x_\tau^E) = V_1(1 - \exp(-V_2 \sum_{\tau=0}^{t-d} x_\tau^E)).$$

We incorporate capacity improvement due to engineering activity into the clearing function using

$$f(\sum_{\tau=0}^{t-d} x_\tau^E, W_\tau) = [K_1 + \phi(\sum_{\tau=0}^{t-d} x_\tau^E)](1 - \exp(-K_2 W_t)).$$

The discrete version of the problem ([13]) on the basis of which we write the optimal control problem version is given by

$$(\mathcal{P}) \quad \max\{\sum_t \mu_t(D_t + S_{t-1} - S_t)$$
$$-(\phi_t^P x_t^P + \omega_t W_t + h_t I_t + \rho_t R_t + l_t S_t + \phi_t^E x_t^E)\}$$

subject to

$$\begin{aligned} W_t &\leq W_{t-1} - x_t^P + R_t, \\ I_t &\leq I_{t-1} + x_t^P - D_t - S_{t-1} + S_t, \\ \xi_t^P x_t^P + \xi_t^E x_t^E &\leq f(\sum_{\tau=0}^{t-d} x_\tau^E, W_t), \\ x_t^P &\leq \gamma_t x_t^P, \\ x_t^P, x_t^E, W_t, R_t, S_t, I_t &\geq 0. \end{aligned} \tag{1}$$

3. CONTROL PROBLEM VERSION

The control problem of interest is now

$$(\mathcal{P}) \quad \max\{\int_0^{t_1} \mu D - (h \max\{0, I(t)\}$$
$$+ l \max\{0, -I(t) + \phi^P x^P + \omega W_t + hI + \rho R + \phi^E x^E) ds\}$$

subject to

$$\begin{aligned} \frac{dW}{dt} &= R - x^P, \\ \frac{dI}{dt} &= x^P - D, \end{aligned} \tag{2}$$

where

$$\xi^P x^P(t) + \xi^E x^E(t) = K_1 + V_1[1 - e^{-V_2 \int_0^t x^E(s)ds}][1 - e^{-K_2 W}],\ x^E \geq 0,\ t > 0, \tag{3}$$

and we take, for now,

$$\xi^P = 1. \tag{4}$$

Then,

$$x^P(t) = -\xi^E x^E(t) + K_1(t) + V_1[1 - e^{-V_2 \int_0^t x^E(s)ds}][1 - e^{-K_2 W}],\ x^E \geq 0,\ t > 0.$$

For the cost integrand we consider

$$\begin{aligned} F(W, I, x^P, x^E) &= \mu D(t) - h \max\{0, I(t)\} - l \max\{0, -I(t)\} \\ &\quad -\omega W(t) - \rho R - \phi^E x^E(t) - \phi^P x^P(t). \end{aligned} \tag{5}$$

where W, I are the states and and x^E engineering control. We may consider different versions of our control. The state $W(t)$ represents inventory of production lots at time t with unit holding cost ω. The state I represents finished goods inventory at the end of period t with corresponding unit cost h. Next R represents number of production lots released at time t with corresponding unit cost ρ. Next, x^P represents output of production at time t with corresponding unit cost ϕ^P, and x^E , which is part of control function, represents number of engineering lots processed at time t with corresponding unit cost ϕ^E. The quantities x^E and $K_1(t)$ play competing rules in determining the output of production. Thus, we might consider K_1 as part of control variable. The quantity S represents number of production lots backordered at time t with corresponding unit cost l, and R represents number of production lots released at time t with corresponding unit cost ρ. Thus, we will take R to be as our third and final control. The quantity D represents demand of production at time t with corresponding unit sale price $\mu(t)$. ξ^P represents capacity consumption factor of processing a production lot in period t, which we will assume to be equal to 1. Finally ξ^E represents consumption factor of processing an engineering lot at time t, which we shall assume to be greater than 1, representing the fact that engineering lots are generally more resource-intensive to process than regular production lots. r In what follows we will add the following constraints to the problem stated above,

$$I(t_1) - W(t_1) \leq 0,$$

$$I(t_1) \quad = \quad 0. \tag{6}$$

We will consider a practical way of solving the above problem including the restriction on the states. Let

$$\begin{aligned} F(W, I, x^P, x^E) \quad = \quad & \mu D(t) - h\max\{0, I(t)\} - l\max\{0, -I(t)\} \\ & -\omega W(t) - \rho R - \phi^E x^E(t) - \phi^P x^P(t). \end{aligned} \tag{7}$$

As mentioned above the controls we consider are $R(t), x^E(t), K_1(t)$. We would like to maximize

$$\int_0^{t_1} F(W, I, x^P, x^E)ds$$

and minimize

$$\int_0^{t_1} \frac{1}{2}[R^2(t) + x^E(t) + K_1(t)^2]dt.$$

Alternatively we will consider the problem of minimizing the weighted quantity

$$\alpha \int_0^{t_1} -F(W, I, x^P, x^E)ds + \beta \int_0^{t_1} \frac{1}{2}[R^2(t) + x^E(t) + K_1(t)^2]dt \tag{8}$$

where

$$\alpha \geq 0, \ \beta \geq 0, \ \alpha + \beta = 1.$$

4. NUMERICAL APPROXIMATION PROCEDURE

We numerically solve the control problem as follows. To deal with the constraints on the states W and I we use penalty method. That is, we update the cost (9) by adding to it

$$\begin{aligned} L(W, I; P_1, ..., P_5, \epsilon_1, \epsilon_2) = 0.5P_1(1 + \frac{2}{\pi})tan^{-1}(\frac{-W}{\epsilon_1}) \\ + 0.5P_2(1 + \frac{2}{\pi})tan^{-1}(\frac{I - W}{\epsilon_2}) + P_4 I^2 + 0.5P_5(1 + \frac{2}{\pi}tan^{-1}(\frac{-I}{\epsilon_3})), \end{aligned}$$

where $P_1, P_2, \cdots P_5$ are penalty parameters to be chosen as appropriate large positive numbers, and $\epsilon_1, \epsilon_2, \epsilon_3$ are also penalty parameters to be chosen to be small positive numbers. We then take the negative of the modified objective function and minimize the corresponding Hamiltonian as a function of the control using the method of descent ([14], [16], [18]). The procedure is to randomly generate an approximation to the controls. Then, we generate the state trajectories, followed by the adjoint variables. The next step is to compute the gradient of the Hamiltonian with respect to the control variables and update the controls, as in conjugate gradient method in nonlinear programming, and continue the process until stopping criteria are met. That is,

- generate random discrete approximations to the controls

$$u(t) = (x^E(t), K_1(t), R(t)),$$

  use Hamiltonian to improve control,

- use $x^E(t), K_1(t), R(t)$ to get the state variables $x = (W, I)$ equation from $t = 0$ to $t = 1$ with initial conditions,

- using the controls and the state variable in the previous steps to determine the costate variables, $\lambda = (\lambda_1, \lambda_2)$ by integrating the costate equations backward from $t = 1$ to $t = 0$,

- update the control: $u = u - \delta \frac{\partial}{\partial u} H(x, p, u, t)$.

  The penalized state and costate equations are given by

$$\begin{aligned}
\frac{dW}{dt} =& R - x^P, \\
\frac{dI}{dt} =& x^P - D, \\
\frac{d\lambda_1}{dt} =& \partial_W(-\alpha F(W, I, x^P, x^E) + \beta \frac{1}{2}[R^2(t) + x^E(t) + K_1(t)^2] \\
& + L(W, I; P_1, ..., P_5, \epsilon_1, \epsilon_2)), \\
\frac{d\lambda_2}{dt} =& \partial_I(-\alpha F(W, I, x^P, x^E) + \beta \frac{1}{2}[R^2(t) + x^E(t) + K_1(t)^2] \\
& + L(W, I; P_1, ..., P_5, \epsilon_1, \epsilon_2)),
\end{aligned} \tag{9}$$

  and the Hamiltonian H is given by

$$\begin{aligned}
H \;=\; & -\alpha F(W, I, x^P, x^E) + \beta \frac{1}{2}[R^2(t) + x^E(t) + K_1(t)^2] \\
& + L(W, I; P_1, ..., P_5, \epsilon_1, \epsilon_2) \\
& - \lambda_1(R - x^P) - \lambda_2(x^P - D),
\end{aligned}$$

$$x^P = -\xi^E x^E(t) + K_1(t) + V_1[1 - e^{-V_2 \int_0^t x^E(s)ds}][1 - e^{-K_2 W}],$$
$$x^E \geq 0,\ t > 0, \tag{10}$$

and $x^E(t), K_1(t), R(t)$ are the control variables.

· terminate the iterative procedure when a stopping criteria is met.
Plot the control variables, demand function, state variables and the cost. In plotting the cost we use the cost in (5).

5. EXAMPLES

5.1. Example 1

We use the time interval $[0, 1]$, where we divided it into 100 subintervals. We used two penalty parameters of 10 and 1000. Next, $\alpha = .695$, and $\beta = .305$. These parameters are used as weighing factors in the two objective functions. $K2 = 0.05, VV1 = 0.2 \cdot 75, VV2 = 0.05$ are parameters appearing in the definition of the clearing function. K_1 represents an initial capacity and K_2 represents a value of steepness of clearing function, that is an indicator of the degree of variability in the production system. The parameter $VV1$ denotes the maximum additional capacity that can be achieved through engineering activity, while $VV2$ is a parameter that controls the rate at which engineering activity improves capacity, which in practice would be governed by the skill of the engineering group responsible. Decreasing returns are expected to scale on engineering activity, where the marginal improvement in capacity from running an additional engineering lot is monotonically decreasing the number of engineering lots processed. Other functional parts in the model were $K1(t), R(t)$, that were initially set to be 100, $\xi^E(t) = 1,\ l(t) = 1.5$, $R(t) = 1,\ \phi^P(t) = 10$. The parameters ξ^E and ϕ^P represent capacity consumption factors of processing an engineer lot and production lot in period t respectively. $R(t)$ represents number of production lots released in period t with corresponding unit cost $\rho(t)$. Further, we have the functional parameter $\mu(t) = 144(9.8)^t$ representing unit sale price of the demand. The demand, in this example is given by

$$D(t) = 20 + 100(1 - exp(0.1t)) - 0.25t^2 + 10t + 20,\ 0 \leq t \leq .75,$$

$$= \quad -0.25t^2 + 10t + 20, \;\; .75 \leq t \leq 1.$$

Simulations are presented below.

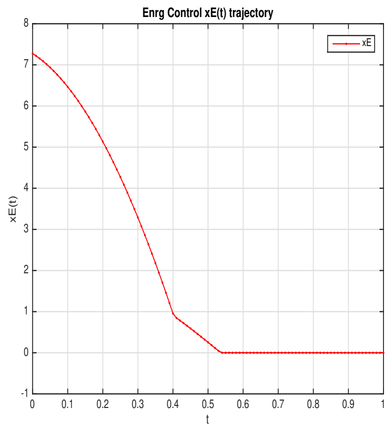

Figure 1: Engineering Control

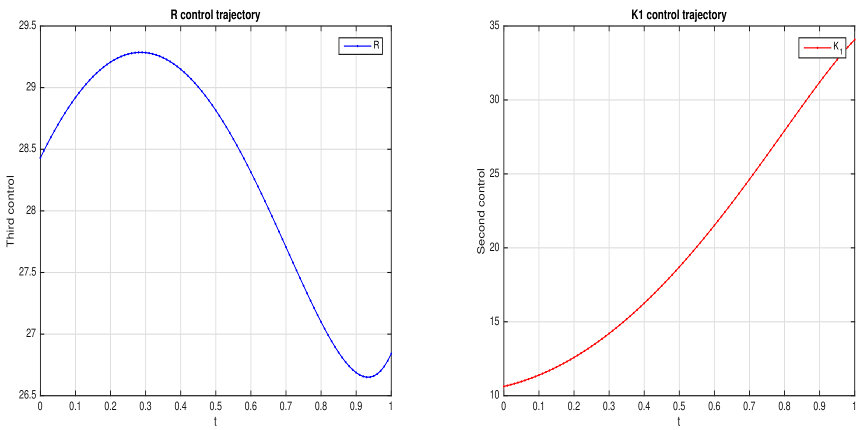

Figure 2: Controls R *and* K_1

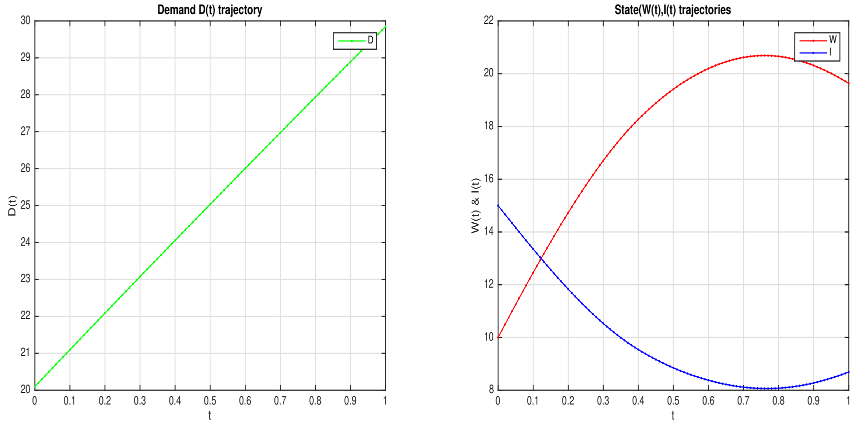

Figure 3: Demand and Work in process and Inventory

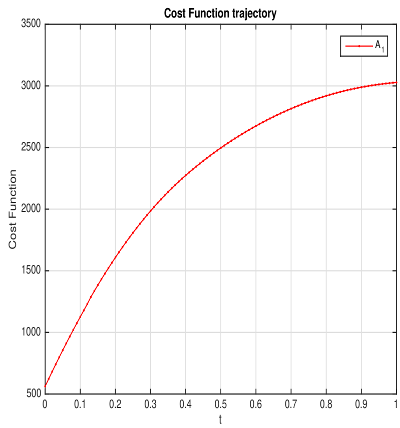

Figure 4: Cost Function

5.2. Example 2

Next, we present a second example where the demand function is given by

$$D(t) = -0.25t^2 + 10t + 20.$$

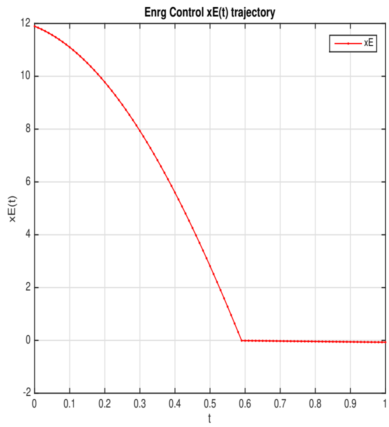

Figure 5: Engineering Control

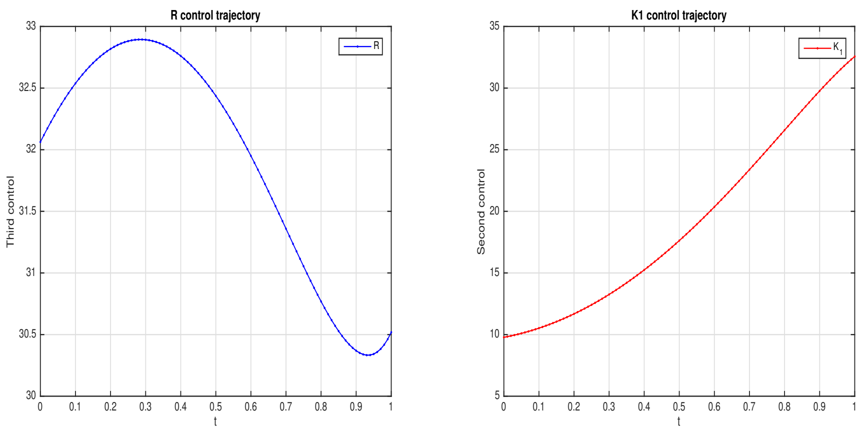

Figure 6: Controls R *and* K_1

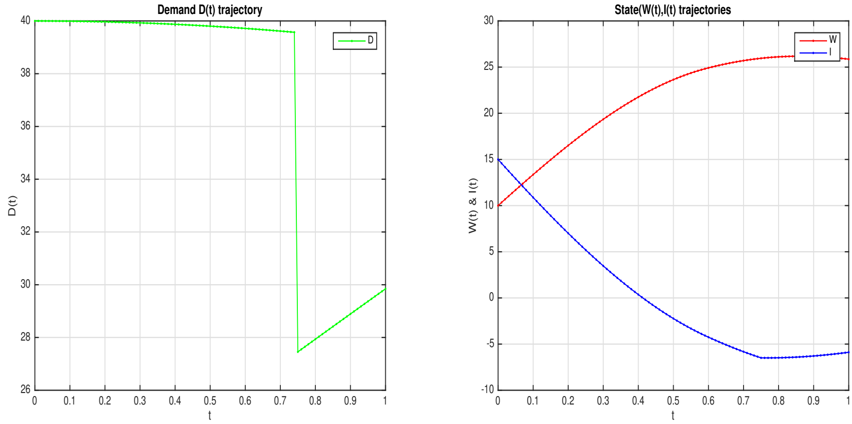

Figure 7: Demand, Work in Process and Inventory

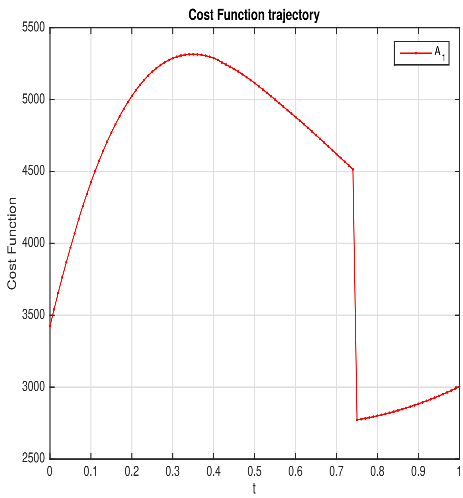

Figure 8: Cost Function

6. DISCUSSION OF THE RESULTS

We used Pontryagin's maximum principle and numerically minimized the Hamiltonian with the respect to the controls. The numerical computations show that the objective functions are optimized and the constraints are satisfied. We note, in particular that inventory at the end of the production cycle is small compared to what it is at the beginning of the cycle in each example. We also note the engineering control is phased out as it should, and the cost function is increasing and tends to level off toward the end. Again, this is reasonable to expect.

7. CONCLUSION

We considered a problem where capacity allocation between production and engineering process improvement play a competing role. A concave clearing function reflecting output production decrease due to competition from new product development, and capacity improvement due to learning effect is used. An appropriate continuous optimal control model is formulated and numerical computations are carried out. This model can be extended to deal with multiproduct model where capacity competition occurs between engineering activities related to different products and between products.

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