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APPLICATION OF A NEW CLASS CUMULATIVE LIFETIME DISTRIBUTION TO SOFTWARE RELIABILITY ANALYSIS

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ABSTRACT: In this paper we consider the application of a a new class of cumulative distribution function proposed by Ramos, Dey, Louzada and Lachos in [9] to the debugging theory.

We study the Hausdorff approximation of the shifted Heaviside step function by this family.

Numerical examples, illustrating our results are presented using programming environment Mathematica.

We give also real examples with data provided in [30] using the new software reliability model. Dataset included [31] Year 2000 compatibility modifications, operating system upgrade, and signaling message processing.

AMS Subject Classification: 68N30, 41A46

Key Words: extended Poisson–exponential cumulative distribution function (EPcdf), Heaviside function, Hausdorff approximation, upper and lower bounds

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1. INTRODUCTION

Some extensions of the well-known Poisson, Poisson–exponential, Chen, Exponentiated Chen, modified Weibull and Burr distributions can be found in: [1]–[8].

Some software reliability models, can be found in [10]–[28].

Another model that uses the "Gompertz–type correction" is the extended Poisson–exponential cumulative distribution function (EPcdf). The (EPcdf) is given by (see for instance [9]):

$$M(t;\lambda;\theta) = \frac{e^{\lambda \frac{1-e^{-\theta t}}{1-e^{-\theta}}} - 1}{e^{\lambda} - 1} \tag{1}$$

where $\theta > 0$; $\lambda > 0$.

We consider the following class of this family with application to the population dynamics and debugging theory:

$$M_1(t) = \frac{e^{\lambda \frac{1-e^{-\theta t}}{1-e^{-\theta}}} - 1}{e^{\lambda} - 1}, \tag{2}$$

with

$$t_0 = -\frac{1}{\theta} \ln \left(1 - \frac{1-e^{-\theta}}{\lambda} \ln \left(\frac{1+e^{\lambda}}{2} \right) \right); \quad M_1(t_0) = \frac{1}{2}. \tag{3}$$

In this note we study the Hausdorff approximation of the *shifted Heaviside step function*

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0, \end{cases}$$

by this family.

Definition 1. [29] The Hausdorff distance (the H–distance) $\rho(f,g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B|| \},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|, |x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

We propose a software modules (intellectual properties) within the programming environment CAS Mathematica for the analysis. The models have been tested with real-world data.

2. MAIN RESULTS

The one–sided Hausdorff distance d between the function $h_{t_0}(t))$ and the function - ((2)–(3)) satisfies the relation

$$M_1(t_0 + d) = 1 - d. \tag{4}$$

The following theorem gives upper and lower bounds for d

Theorem 1. Let

$$p = -\frac{1}{2},$$

$$q = 1 + \frac{\theta\lambda(1+e^{\lambda})}{2(1-e^{-\theta})(e^{\lambda}-1)}\left(1 - \frac{1-e^{-\theta}}{\lambda}\ln\frac{1+e^{\lambda}}{2}\right),$$

$$r = 2.1q$$

For the one–sided Hausdorff distance d between $h_{t_0}(t))$ and the function ((2)–(3)) the following inequalities hold for: $q > \frac{e^{1.05}}{2.1}$

$$d_l = \frac{1}{r} < d < \frac{\ln r}{r} = d_r. \tag{5}$$

Proof. Let us examine the function:

$$F(d) = M_1(t_0 + d) - 1 + d. \tag{6}$$

From $F'(d) > 0$ we conclude that function F is increasing.

Consider the function

$$G(d) = p + qd. \tag{7}$$

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.

Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 1).

In addition $G'(d) > 0$.

Further, for $q > \frac{e^{1.05}}{2.1}$ we have $G(d_l) < 0$ and $G(d_r) > 0$.

This completes the proof of the theorem.

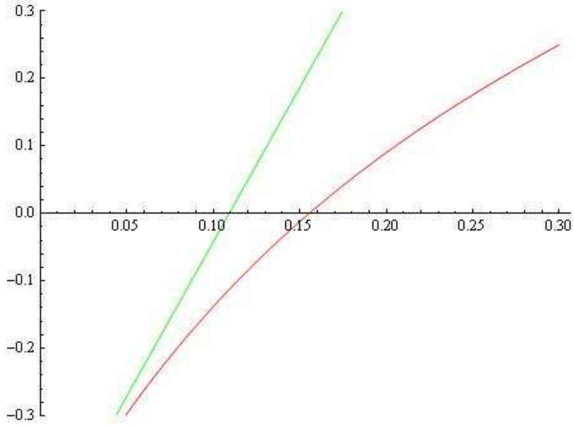

Figure 1: The functions $F(d)$ and $G(d)$ for $\theta = 8; \lambda = 0.5$.

The model ((2)–(3)) for $\theta = 8$, $\lambda = 0.5$, $t_0 = 0.103098$ is visualized on Fig. 2. From the nonlinear equation (4) and inequalities (5) we have: $d = 0.155378$, $d_l = 0.103958$, $d_r = 0.235337$.

The model ((2)–(3)) for $\beta = 15$, $\lambda = 0.01$, $t_0 = 0.0463767$ is visualized on Fig. 3. From the nonlinear equation (4) and inequalities (5) we have: $d = 0.104496$, $d_l = 0.0561458$, $d_r = 0.161689$.

The model ((2)–(3)) for $\beta = 25$, $\lambda = 0.005$, $t_0 = 0.0277759$ is visualized on Fig. 4. From the nonlinear equation (4) and inequalities (5) we have: $d = 0.0756055$, $d_l = 0.0353142$, $d_r = 0.118072$.

From the above examples, it can be seen that the proven estimates (see Theorem 1) for the value of the Hausdorff approximation is reliable when assessing the important characteristic - "saturation".

This characteristic (as we have already shown in our previous publications) has its equal participation together with the other two characteristics - "confidence intervals" and "confidence bounds" in the area of the Software Reliability Theory.

We propose a software module (intellectual properties) within the programming environment *CAS Mathematica* for the analysis of the considered family $M_1(t)$.

The module offers the following possibilities:

- generation of the function under user defined values of the parameters λ,

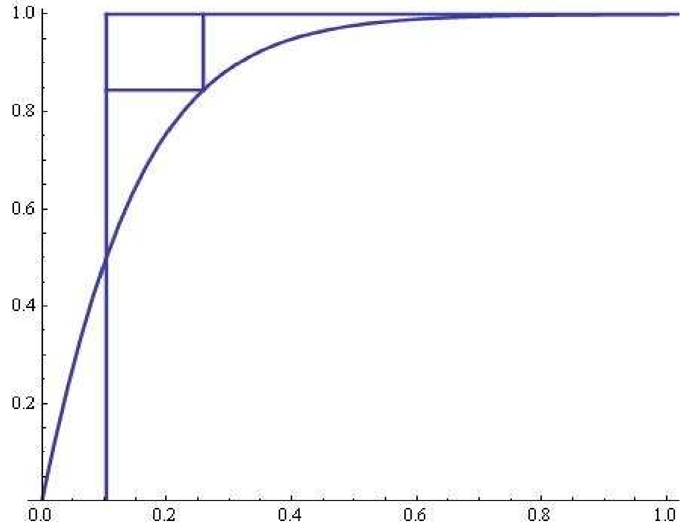

Figure 2: The model ((2)–(3)) for $\theta = 8$, $\lambda = 0.5$, $t_0 = 0.103098$; H–distance $d = 0.155378$, $d_l = 0.103958$, $d_r = 0.235337$.

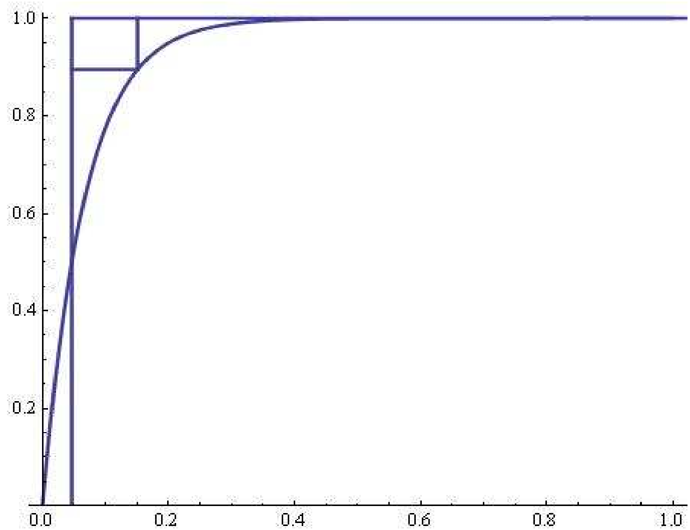

Figure 3: The model ((2)–(3)) for $\beta = 15$, $\lambda = 0.01$, $t_0 = 0.0463767$; H–distance $d = 0.104496$, $d_l = 0.0561458$, $d_r = 0.161689$.

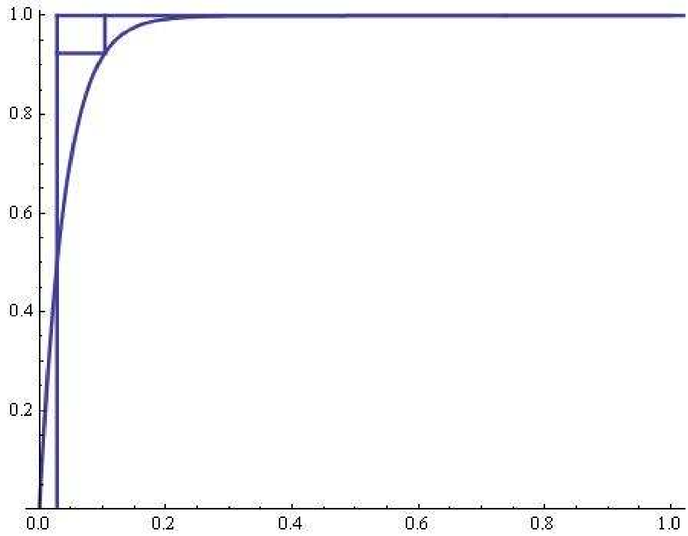

Figure 4: The model ((2)–(3)) for $\beta = 25$, $\lambda = 0.005$, $t_0 = 0.0277759$; H–distance $d = 0.0756055$, $d_l = 0.0353142$, $d_r = 0.118072$.

and θ;

- calculation of the H-distance d between the function $h_{t_0}(t))$ and the function $M_1(t)$;

- software tools for animation and visualization.

2.1. APPLICATION IN THE FIELD OF DEBUGGING AND TEST THEORY

We give real examples with data provided in [30].

The operating time of the software is 167,900 days. 115 failures are detected for these days which contain 71 unique failures.

Table 1 shows the failures data which are united for each of the 13 months.

Dataset included [31] Year 2000 compatibility modifications, operating system upgrade, and signaling message processing.

Month Index	System Days (Days)	System Days (Cumulative)	Failures	Cumulative Failures
1	961	961	7	7
2	4170	5131	3	10
3	8789	13,920	14	24
4	11,858	25,778	8	32
5	13,110	38,888	11	43
6	14,198	53,086	8	51
7	14,265	67,351	7	58
8	15,175	82,526	19	77
9	15,376	97,902	17	94
10	15,704	113,606	6	100
11	18,182	131,788	11	111
12	17,760	149,548	4	115
13	18,352	167,900	0	115

Table 1. Field failure data [30].

The fitted model

$$M_1(t) = \omega \frac{e^{\lambda \frac{1-e^{-\theta t}}{1-e^{-\theta}}} - 1}{e^{\lambda} - 1}$$

based on the data of Table 1 for the estimated parameters:

$$\omega = 7; \;\; \theta = 0.142302; \;\; \lambda = 0.317432$$

is plotted on Fig. 5.

We hope that the results will be useful for specialists in this scientific area.

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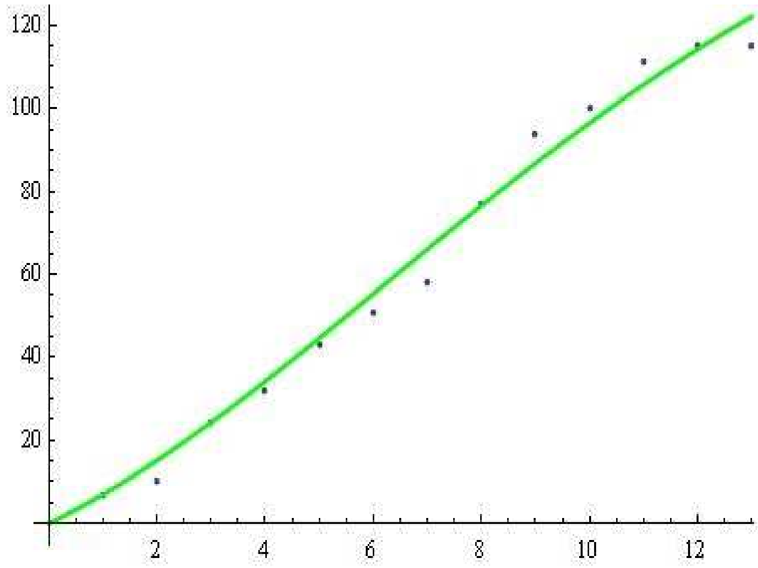

Figure 5: The fitted model $M_1(t)$.

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