Communications in Applied Analysis, **22**, No. 3 (2018), 447-457 ISSN: 1083-2564

EIGHT-FOLDS OF COMPLETE INTERSECTION CALABI-YAU

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ABSTRACT: In paper a complete intersection Calabi-Yau 8-folds are considered. Their Hodge diamond, Todd classes and Chern characters for sheaves of differential k-forms are calculated.

AMS Subject Classification: 14J32, 14J40, 14Q10

Key Words: Calabi-Yau manifolds, Hodge diamond, Chern character

Received: November 23, 2017 ; **Accepted:** June 13, 2018 ;
Published: June 21, 2018. **doi:** 10.12732/caa.v22i3.7
Dynamic Publishers, Inc., Acad. Publishers, Ltd. http://www.acadsol.eu/caa

1. INTRODUCTION

All definitions of this article are conventional [1]. We work in projective space $\mathbb{P}^n$ over an algebraically closed field of arbitrary characteristic. Let x be divisor degree of hypersurface $X \in \mathbb{P}^n$. We denote $X_m = \bigcap_{i=1}^{k} S_i^{s_i} \in \mathbb{P}^n$ as m-fold, which is a complete intersection of k hypersurfaces S_i degree s_i. Then X is a Calabi-Yau if $x = \sum s_i = n+1$. For a sheaf of differential forms $\Omega_X^i = \Lambda^i \Omega_X$ we introduce Hodge numbers $h^{ij} = \dim \mathrm{H^i}(\Omega_X^j)$, which not only are symmetrical: $h^{ij} = h^{ji}$, but Serre symmetrical also: $h^{ij} = h^{n-i,n-j}$. If we have Hodge numbers then Betti numbers may be calculated as

$$b_k = \sum_{i+j=k} h^{ij}.$$

We can also define the Euler characteristic of X as the alternating sum of the Betti numbers:

$$\chi(X) = \sum_k (-)^k b_k.$$

For clarity, we rotate the matrix h^{ij} on 45° and call it as the Hodge diamond. So, for $n = 3$, we will have:

$$\begin{array}{ccccccc} & & & h^{00} & & & \\ & & h^{10} & & h^{01} & & \\ & h^{20} & & h^{11} & & h^{02} & \\ h^{30} & & h^{21} & & h^{12} & & h^{03} \\ & h^{31} & & h^{22} & & h^{13} & \\ & & h^{32} & & h^{23} & & \\ & & & h^{33} & & & \end{array} \qquad \begin{array}{l} b_0 = h^{00} \\ b_1 = h^{10} + h^{01} \\ b_2 = h^{20} + h^{11} + h^{02} \\ b_3 = h^{30} + h^{21} + h^{12} + h^{03} \\ b_4 = h^{31} + h^{22} + h^{13} \\ b_5 = h^{32} + h^{23} \\ b_6 = h^{33} \end{array}$$

If $X = \bigcap_{k=1}^{t} S_k$ is complete intersection of t-hypersurfaces of degree s_k, then $N_X = \bigoplus_{k=1}^{t} \mathcal{O}_X(-s_k)$ and the sheaf $\mathcal{O}_X$ can be determined with used the following recurrence relations:

$$\begin{gathered} 0 - \mathcal{O}_{S_1 \cap S_2 \cap \ldots \cap S_{k-1}}(-s_k) - \mathcal{O}_{S_1 \cap S_2 \cap \ldots \cap S_{k-1}} - \mathcal{O}_{S_1 \cap S_2 \cap \ldots \cap S_k} - 0 \\ 0 - \mathcal{O}_{S_1 \cap S_2 \cap \ldots \cap S_{k-2}}(-s_{k-1}) - \mathcal{O}_{S_1 \cap S_2 \cap \ldots \cap S_{k-2}} - \mathcal{O}_{S_1 \cap S_2 \cap \ldots \cap S_{k-1}} - 0 \\ \ldots \\ 0 - \mathcal{O}_{S_1}(-s_2) - \mathcal{O}_{S_1} - \mathcal{O}_{S_1 \cap S_2} - 0 \\ 0 - \mathcal{O}_{\mathbb{P}^n}(-s_1) - \mathcal{O}_{\mathbb{P}^n} - \mathcal{O}_{S_1} - 0 \end{gathered}$$

Now, for calculations of $H^i(\Omega_X^k)$ we can get the power series

$$0 - Sym^k N_X - Sym^{k-1} N_X \otimes \Omega_{\mathbb{P}^n|X} - \ldots - \Omega^k_{\mathbb{P}^n|X} - \Omega^k_X - 0.$$

2. TODD AND CHERN CLASSES

As shown in [2] for any rational complete intersection Calabi-Yau X all non-zero entries of the Hodge diamond always lying on its equator or on the central column. Also $h^{ii} = 1$ if $i \neq m/2$. Therefore, we can simplify all calculation with use of characteristic classes theory.

We take the Riemann-Roch-Hirzebruch equation

$$\chi(E, X) = \int_X \mathrm{ch(E)} \wedge \mathrm{td(T_X)}, \tag{1}$$

attach it to $E = \bigwedge^q \Omega_X = \Omega_X^q$ and rewrite over Chern classes of tangent bundle $c(T_X) = \sum\limits_i c_i(T_X) = \prod\limits_i (1 + \alpha_i)$:

$$\mathrm{td(T_X)} = \prod_{\mathrm{i}} \frac{\alpha_{\mathrm{i}}}{1 - \mathrm{e}^{-\alpha_{\mathrm{i}}}},$$

$$\mathrm{ch}(\Omega_{\mathrm{X}}^{\mathrm{q}}) = \sum_{\mathrm{i_1 < i_2 < \ldots < i_q}} \mathrm{e}^{\alpha_{\mathrm{i_1}}} \ldots \mathrm{e}^{\alpha_{\mathrm{i_q}}}. \tag{2}$$

Here α_i are Chern roots of $T_{\mathbb{P}^n}$.

For the first time the classes $\mathrm{td(T_X)}$ up to td_6 were obtained in [3] (p.14). In the original paper [4] (p.221) the Eq.(61) coincide with the td_6 in our notation. All classes up to td_9 were obtained in [5], up to td_{10} in [2]. For Chern characters of 8-folds X from (2) we have

$\mathbf{ch}(\Omega_8^1)$:

$\mathrm{ch}_0 = 8; \quad \mathrm{ch}_1 = -c_1; \quad \mathrm{ch}_2 = \frac{1}{2}\left(c_1^2 - 2c_2\right);$

$\mathrm{ch}_3 = \frac{\left(-c_1^3 + 3c_1c_2 - 3c_3\right)}{6}; \quad \mathrm{ch}_4 = \frac{\left(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4\right)}{24};$

$\mathrm{ch}_5 = \frac{\left(-c_1^5 + 5c_1^3c_2 - 5c_1c_2^2 - 5c_1^2c_3 + 5c_2c_3 + 5c_1c_4 - 5c_5\right)}{120};$

$$\mathrm{ch}_6 = \frac{1}{720}\begin{pmatrix} c_1^6 - 6c_1^4c_2 + 9c_1^2c_2^2 - 2c_2^3 + 6c_1^3c_3 - 12c_1c_2c_3 \\ +3c_3^2 - 6c_1^2c_4 + 6c_2c_4 + 6c_1c_5 - 6c_6 \end{pmatrix};$$

$$\mathrm{ch}_7 = \frac{1}{5040}\begin{pmatrix} -c_1^7 + 7c_1^5c_2 - 14c_1^3c_2^2 + 7c_1c_2^3 - 7c_1^4c_3 + 21c_1^2c_2c_3 \\ -7c_2^2c_3 - 7c_1c_3^2 + 7c_1^3c_4 \\ -14c_1c_2c_4 + 7c_3c_4 - 7c_1^2c_5 + 7c_2c_5 + 7c_1c_6 - 7c_7 \end{pmatrix};$$

$$\mathrm{ch}_8=\frac{1}{40320}\begin{pmatrix} c_1^8-8c_1^6c_2+20c_1^4c_2^2-16c_1^2c_2^3+2c_2^4+8c_1^5c_3 \\ -32c_1^3c_2c_3+24c_1c_2^2c_3+12c_1^2c_3^2-8c_2c_3^2-8c_1^4c_4 \\ +24c_1^2c_2c_4-8c_2^2c_4-16c_1c_3c_4+4c_4^2+8c_1^3c_5 \\ -16c_1c_2c_5+8c_3c_5-8c_1^2c_6+8c_2c_6+8c_1c_7-8c_8 \end{pmatrix}.$$

$\mathbf{ch}(\Omega_8^2)$:

$\mathrm{ch}_0=28$; $\mathrm{ch}_1=-7c_1$; $\mathrm{ch}_2=\frac{\left(7c_1^2-12c_2\right)}{2}$;

$\mathrm{ch}_3=\frac{\left(-7c_1^3+18c_1c_2-12c_3\right)}{6}$; $\mathrm{ch}_4=\frac{\left(7c_1^4-24c_1^2c_2+12c_2^2+12c_1c_3\right)}{24}$;

$\mathrm{ch}_5=\frac{\left(-7c_1^5+30c_1^3c_2-30c_1c_2^2-10c_1^2c_3+20c_2c_3-20c_1c_4+40c_5\right)}{120}$;

$$\mathrm{ch}_6=\frac{1}{720}\begin{pmatrix} 7c_1^6-36c_1^4c_2+54c_1^2c_2^2-12c_2^3+6c_1^3c_3-42c_1c_2c_3 \\ +18c_3^2+54c_1^2c_4-24c_2c_4-114c_1c_5+144c_6 \end{pmatrix};$$

$$\mathrm{ch}_7=\frac{1}{720}\begin{pmatrix} -c_1^7+6c_1^5c_2-12c_1^3c_2^2+6c_1c_2^3+9c_1^2c_2c_3 \\ -4c_2^2c_3-7c_1c_3^2-15c_1^3c_4+16c_1c_2c_4+4c_3c_4 \\ +35c_1^2c_5-26c_2c_5-50c_1c_6+56c_7 \end{pmatrix};$$

$$\mathrm{ch}_8=\frac{1}{40320}\begin{pmatrix} 7c_1^8-48c_1^6c_2+120c_1^4c_2^2-96c_1^2c_2^3+12c_2^4-8c_1^5c_3 \\ -80c_1^3c_2c_3+88c_1c_2^2c_3+100c_1^2c_3^2-48c_2c_3^2 \\ +176c_1^4c_4-304c_1^2c_2c_4+64c_2^2c_4-96c_1c_3c_4 \\ +80c_4^2-456c_1^3c_5+688c_1c_2c_5-120c_3c_5 \\ +736c_1^2c_6-624c_2c_6-904c_1c_7+960c_8 \end{pmatrix}.$$

$\mathbf{ch}(\Omega_8^3)$:

$\mathrm{ch}_0=56$; $\mathrm{ch}_1=-21c_1$; $\mathrm{ch}_2=\frac{3}{2}\left(7c_1^2-10c_2\right)$;

$\mathrm{ch}_3=\frac{\left(-7c_1^3+15c_1c_2-5c_3\right)}{2}$; $\mathrm{ch}_4=\frac{\left(7c_1^4-20c_1^2c_2+10c_2^2+12c_4\right)}{8}$;

$\mathrm{ch}_5=\frac{\left(-21c_1^5+75c_1^3c_2-75c_1c_2^2+25c_1^2c_3+25c_2c_3-115c_1c_4+95c_5\right)}{120}$;

$$\mathrm{ch}_6=\frac{1}{240}\begin{pmatrix} 7c_1^6-30c_1^4c_2+45c_1^2c_2^2-10c_2^3-20c_1^3c_3-10c_1c_2c_3 \\ +15c_3^2+80c_1^2c_4-50c_2c_4-60c_1c_5-30c_6 \end{pmatrix};$$

$$\mathrm{ch}_7=\frac{1}{720}\begin{pmatrix} -3c_1^7+15c_1^5c_2-30c_1^3c_2^2+15c_1c_2^3+15c_1^4c_3 \\ -5c_2^2c_3-20c_1c_3^2-60c_1^3c_4+80c_1c_2c_4 \\ +5c_3c_4+40c_1^2c_5-55c_2c_5+95c_1c_6-245c_7 \end{pmatrix};$$

$$\mathrm{ch}_8 = \frac{\left(\begin{array}{c} 21c_1^8 - 120c_1^6c_2 + 300c_1^4c_2^2 - 240c_1^2c_2^3 \\ +30c_2^4 - 160c_1^5c_3 + 80c_1^3c_2c_3 + 80c_1c_2^2c_3 \\ +320c_1^2c_3^2 - 120c_2c_3^2 + 664c_1^4c_4 - 1376c_1^2c_2c_4 + 328c_2^2c_4 \\ -184c_1c_3c_4 + 284c_4^2 - 384c_1^3c_5 + 1272c_1c_2c_5 - 552c_3c_5 \\ -2136c_1^2c_6 + 792c_2c_6 + 6336c_1c_7 - 9528c_8 \end{array}\right)}{40320}.$$

$\mathbf{ch}(\Omega_8^4)$: $\mathrm{ch}_0 = 70; \mathrm{ch}_1 = -35c_1;\quad \mathrm{ch}_2 = \frac{5}{2}\left(7c_1^2 - 8c_2\right);$

$$\mathrm{ch}_3 = -\frac{5}{6}\left(7c_1^3 - 12c_1c_2\right);\quad \mathrm{ch}_4 = \frac{\left(35c_1^4 - 80c_1^2c_2 + 40c_2^2 - 40c_1c_3 + 64c_4\right)}{24};$$

$$\mathrm{ch}_5 = \frac{\left(-7c_1^5 + 20c_1^3c_2 - 20c_1c_2^2 + 20c_1^2c_3 - 32c_1c_4\right)}{24};$$

$$\mathrm{ch}_6 = \frac{\left(\begin{array}{c} 7c_1^6 - 24c_1^4c_2 + 36c_1^2c_2^2 - 8c_2^3 - 36c_1^3c_3 + 12c_1c_2c_3 \\ +12c_3^2 + 60c_1^2c_4 - 48c_2c_4 + 36c_1c_5 - 96c_6 \end{array}\right)}{144};$$

$$\mathrm{ch}_7 = \frac{\left(\begin{array}{c} -c_1^7 + 4c_1^5c_2 - 8c_1^3c_2^2 + 4c_1c_2^3 + 8c_1^4c_3 \\ -6c_1^2c_2c_3 - 6c_1c_3^2 - 14c_1^3c_4 \\ +24c_1c_2c_4 - 18c_1^2c_5 + 48c_1c_6 \end{array}\right)}{144};$$

$$\mathrm{ch}_8 = \frac{\left(\begin{array}{c} 35c_1^8 - 160c_1^6c_2 + 400c_1^4c_2^2 - 320c_1^2c_2^3 + 40c_2^4 \\ -400c_1^5c_3 + 480c_1^3c_2c_3 - 80c_1c_2^2c_3 + 520c_1^2c_3^2 \\ -160c_2c_3^2 + 736c_1^4c_4 - 1984c_1^2c_2c_4 + 512c_2^2c_4 \\ -96c_1c_3c_4 + 416c_4^2 + 1504c_1^3c_5 - 432c_1c_2c_5 - 848c_3c_5 \\ -4304c_1^2c_6 + 2848c_2c_6 - 2416c_1c_7 + 19328c_8 \end{array}\right)}{40320}.$$

$\mathbf{ch}(\Omega_8^5)$:

$\mathrm{ch}_0 = 56;\quad \mathrm{ch}_1 = -35c_1;\quad \mathrm{ch}_2 = \frac{5}{2}\left(7c_1^2 - 6c_2\right);$

$$\mathrm{ch}_3 = -\frac{5\left(7c_1^3 - 9c_1c_2 - 3c_3\right)}{6};\quad \mathrm{ch}_4 = \frac{\left(35c_1^4 - 60c_1^2c_2 + 30c_2^2 - 60c_1c_3 + 36c_4\right)}{24};$$

$$\mathrm{ch}_5 = \frac{\left(-7c_1^5 + 15c_1^3c_2 - 15c_1c_2^2 + 25c_1^2c_3 - 5c_2c_3 - 13c_1c_4 - 19c_5\right)}{24};$$

$$\mathrm{ch}_6 = \frac{\left(\begin{array}{c} 7c_1^6 - 18c_1^4c_2 + 27c_1^2c_2^2 - 6c_2^3 - 42c_1^3c_3 \\ +24c_1c_2c_3 + 9c_3^2 + 18c_1^2c_4 - 30c_2c_4 + 78c_1c_5 - 18c_6 \end{array}\right)}{144};$$

$$\mathrm{ch}_7 = \frac{\begin{pmatrix} -c_1^7 + 3c_1^5c_2 - 6c_1^3c_2^2 + 3c_1c_2^3 + 9c_1^4c_3 - 9c_1^2c_2c_3 \\ +c_2^2c_3 - 5c_1c_3^2 - 3c_1^3c_4 + 14c_1c_2c_4 \\ -c_3c_4 - 29c_1^2c_5 + 11c_2c_5 - c_1c_6 + 49c_7 \end{pmatrix}}{144};$$

$$\mathrm{ch}_8 = \frac{\begin{pmatrix} 35c_1^8 - 120c_1^6c_2 + 300c_1^4c_2^2 - 240c_1^2c_2^3 + 30c_2^4 \\ -440c_1^5c_3 + 640c_1^3c_2c_3 - 200c_1c_2^2c_3 + 460c_1^2c_3^2 \\ -120c_2c_3^2 + 104c_1^4c_4 - 1096c_1^2c_2c_4 + 328c_2^2c_4 \\ +96c_1c_3c_4 + 284c_4^2 + 2136c_1^3c_5 - 1808c_1c_2c_5 \\ -552c_3c_5 + 664c_1^2c_6 + 792c_2c_6 - 7384c_1c_7 - 9528c_8 \end{pmatrix}}{40320}.$$

$\mathbf{ch}(\Omega_8^6)$:

$\mathrm{ch}(0)=28$; $\mathrm{ch}_1 = -21c_1$; $\mathrm{ch}_2 = \frac{3}{2}\left(7c_1^2 - 4c_2\right)$;

$\mathrm{ch}_3 = \frac{\left(-7c_1^3+6c_1c_2+4c_3\right)}{2}$; $\mathrm{ch}_4 = \frac{\left(7c_1^4-8c_1^2c_2+4c_2^2-12c_1c_3\right)}{8}$;

$\mathrm{ch}_5 = \frac{\left(-21c_1^5+30c_1^3c_2-30c_1c_2^2+70c_1^2c_3-20c_2c_3+20c_1c_4-40c_5\right)}{120}$;

$$\mathrm{ch}_6 = \frac{\begin{pmatrix} 7c_1^6 - 12c_1^4c_2 + 18c_1^2c_2^2 - 4c_2^3 - 38c_1^3c_3 \\ +26c_1c_2c_3 + 6c_3^2 - 22c_1^2c_4 - 8c_2c_4 + 42c_1c_5 + 48c_6 \end{pmatrix}}{240};$$

$$\mathrm{ch}_7 = \frac{\begin{pmatrix} -3c_1^7 + 6c_1^5c_2 - 12c_1^3c_2^2 + 6c_1c_2^3 + 24c_1^4c_3 \\ -27c_1^2c_2c_3 + 4c_2^2c_3 - 11c_1c_3^2 + 21c_1^3c_4 + 8c_1c_2c_4 \\ -4c_3c_4 - 41c_1^2c_5 + 26c_2c_5 - 94c_1c_6 - 56c_7 \end{pmatrix}}{720};$$

$$\mathrm{ch}_8 = \frac{\begin{pmatrix} 21c_1^8 - 48c_1^6c_2 + 120c_1^4c_2^2 - 96c_1^2c_2^3 \\ +12c_2^4 - 232c_1^5c_3 + 368c_1^3c_2c_3 - 136c_1c_2^2c_3 \\ +212c_1^2c_3^2 - 48c_2c_3^2 - 272c_1^4c_4 - 80c_1^2c_2c_4 \\ +64c_2^2c_4 + 128c_1c_3c_4 + 80c_4^2 + 552c_1^3c_5 - 768c_1c_2c_5 \\ -120c_3c_5 + 1968c_1^2c_6 - 624c_2c_6 + 2232c_1c_7 + 960c_8 \end{pmatrix}}{40320}.$$

$\mathbf{ch}(\Omega_8^7)$:

$\mathrm{ch}_0 = 8$; $\mathrm{ch}_1 = -7c_1$; $\mathrm{ch}_2 = \frac{1}{2}\left(7c_1^2 - 2c_2\right)$;

$\mathrm{ch}_3 = \frac{\left(-7c_1^3+3c_1c_2+3c_3\right)}{6}$; $\mathrm{ch}_4 = \frac{\left(7c_1^4-4c_1^2c_2+2c_2^2-8c_1c_3-4c_4\right)}{24}$;

$$\mathrm{ch}_5 = \tfrac{1}{120}\left(-7c_1^5 + 5c_1^3c_2 - 5c_1c_2^2 + 15c_1^2c_3 - 5c_2c_3 + 15c_1c_4 + 5c_5\right);$$

$$\mathrm{ch}_6 = \frac{\begin{pmatrix} 7c_1^6 - 6c_1^4c_2 + 9c_1^2c_2^2 - 2c_2^3 - 24c_1^3c_3 \\ +18c_1c_2c_3 + 3c_3^2 - 36c_1^2c_4 + 6c_2c_4 - 24c_1c_5 - 6c_6 \end{pmatrix}}{720};$$

$$\mathrm{ch}_7 = \frac{\begin{pmatrix} -c_1^7 + c_1^5c_2 - 2c_1^3c_2^2 + c_1c_2^3 + 5c_1^4c_3 \\ -6c_1^2c_2c_3 + c_2^2c_3 - 2c_1c_3^2 + 10c_1^3c_4 - 4c_1c_2c_4 \\ -c_3c_4 + 10c_1^2c_5 - c_2c_5 + 5c_1c_6 + c_7 \end{pmatrix}}{720};$$

$$\mathrm{ch}_8 = \frac{\begin{pmatrix} 7c_1^8 - 8c_1^6c_2 + 20c_1^4c_2^2 - 16c_1^2c_2^3 + 2c_2^4 - 48c_1^5c_3 \\ +80c_1^3c_2c_3 - 32c_1c_2^2c_3 + 40c_1^2c_3^2 - 8c_2c_3^2 - 120c_1^4c_4 \\ +80c_1^2c_2c_4 - 8c_2^2c_4 + 40c_1c_3c_4 + 4c_4^2 - 160c_1^3c_5 \\ +40c_1c_2c_5 + 8c_3c_5 - 120c_1^2c_6 + 8c_2c_6 - 48c_1c_7 - 8c_8 \end{pmatrix}}{40320}.$$

$\mathbf{ch}(\Omega_8^8)$:

$$\mathrm{ch}_0 = 1;\quad \mathrm{ch}_1 = -c_1;\quad \mathrm{ch}_2 = \tfrac{c_1^2}{2};\quad \mathrm{ch}_3 = -\tfrac{c_1^3}{6};$$
$$\mathrm{ch}_4 = \tfrac{c_1^4}{24};\quad \mathrm{ch}_5 = -\tfrac{c_1^5}{120};\quad \mathrm{ch}_6 = \tfrac{c_1^6}{720};\quad \mathrm{ch}_7 = -\tfrac{c_1^7}{5040};\quad \mathrm{ch}_8 = \tfrac{c_1^8}{40320}.$$

3. HODGE DIAMOND

The Hodge diamond of 2-fold Calabi-Yau may be see in [1] (p.590), for 3-folds in [6] (p.45). Four-folds was considered in [7], five-folds in [8], six-folds in [2] and 7-folds in [9].

For Example we find the Hodge diamond of 8-fold Calabi-Yau $X_8 \in \mathbb{P}^{12}$, which is a complete intersection of two quadrics one cubic and one sextic $X_8 = S^6 \cap S^3 \cap S^2 \cap S^2$:

$$
\begin{array}{ccccccccc}
 & & & & & & & & 1 \\
 & & & & & & & 0 & \\
 & & & & & & 0 & & 1 \\
 & & & & & 0 & & 0 & \\
 & & & & 0 & & 0 & & 1 \\
 & & & 0 & & 0 & & 0 & \\
 & & 0 & & 0 & & 0 & & 1 \\
 & 0 & & 0 & & 0 & & 0 & \\
1 & & 15023 & & 1218340 & & 11938339 & & 24497442 \\
 & 0 & & 0 & & 0 & & 0 & \\
 & & 0 & & 0 & & 0 & & 1 \\
 & & & 0 & & 0 & & 0 & \\
 & & & & 0 & & 0 & & 1 \\
 & & & & & 0 & & 0 & \\
 & & & & & & 0 & & 1 \\
 & & & & & & & 0 & \\
 & & & & & & & & 1
\end{array}
$$

Summing the Betti numbers

$$\mathbf{b} = (1, 0, 1, 0, 1, 0, 1, 0, 50840848, 0, 1, 0, 1, 0, 1, 0, 1)$$

we obtain $\chi = 50840856$. $\triangle$

This result is easily verified using the theory of characteristic classes. For any morphism $X \xrightarrow{f} \mathbb{P}^n$ with uses the bundle

$$0 - T_X - f^* T_{\mathbb{P}^n} - N_f - 0 \tag{3}$$

we find Chern class $c(T_X)$. Defining $c(\mathcal{O}_{\mathbb{P}^n}(d))) = 1 + dt$, for a complete intersection k hypersurfaces $X = \bigcap_{i=1}^{k} S_i$ of degree s_i we obtain $c(N_f) = \prod_{i=1}^{k}(1+ s_i t)$. The Euler sequence dual to (3) has the form

$$0 - \mathcal{O}_{\mathbb{P}^n} - \mathcal{O}_{\mathbb{P}^n}(1)^{\oplus(n+1)} - T_{\mathbb{P}^n} - 0.$$

From this $c(T_{\mathbb{P}^n}) = (1 + t)^{n+1}$, hense

$$c(T_X) = \frac{(1 + t)^{n+1}}{\prod_{i=1}^{k}(1 + s_i t)}.$$

The Euler characteristic of m-fold is

$$\chi = \int_X c_m(X) = \int_{\mathbb{P}^n} c_{n-m}(N_f) \wedge c_m(T_X) = c_{n-m}(N_f) \cdot c_m(T_X). \qquad (4)$$

For the intersection of $X_8 = S^2 \cap S^2 \cap S^3 \cap S^6 \in \mathbb{P}^{12}$ we have

$$\begin{aligned} c(N_f) &= (1+2t)^2(1+3t)(1+6t) \\ &= 1+13t+58t^2+108t^3+72t^4, \quad \text{i.e.} \quad c_4(N_f) = 72 \\ c(T_X) &= \frac{(1+t)^{13}}{((1+2t)^2(1+3t)(1+6t)} \\ &\approx 1+20t^2+...+706123t^8+..., \quad \Rightarrow \quad c_8(T_X) = 706123. \end{aligned}$$

Therefore the Euler characteristic from (4) has the same value χ:

$$\chi = \int_X c_8(X) = \int_{\mathbb{P}^{12}} c_4(N_f) \wedge c_8(T_X) = 50840856. \qquad \triangle$$

In the following table we show the Hodge numbers of the 8-fold Calabi-Yay that are complete intersections in ordinary projective spaces.

N	$\mathbb{P}^n \vert X_8$	h^{17}	h^{26}	h^{35}	h^{44}	χ
1	[9|10]	92 278	8 337 880	82 044 082	167 729 960	348 678 450
2	[10|92]	72 874	6 522 220	64 153 990	131 198 168	272 696 346
3	[10|83]	40 919	3 549 019	34 868 415	71 381 708	148 298 424
4	[10|74]	20 041	1 652 905	16 208 257	33 238 460	69 000 876
5	[10|65]	10 878	855 130	8 367 182	17 190 560	35 656 950
6	[11|822]	52 203	4 599 355	45 212 739	92 511 492	192 240 096
7	[11|732]	26 453	2 233 660	21 922 539	44 920 592	93 285 906
8	[11|642]	12 153	965 965	9 457 229	19 420 832	40 291 536
9	[11|633]	12 229	974 314	9 540 535	19 589 528	40 643 694
10	[11|552]	7 938	608 380	5 945 742	12 228 120	25 352 250
11	[11|543]	5 849	435 865	4 254 993	8 759 996	18 153 420
12	[11|444]	3 943	282 079	2 748 151	5 667 644	11 736 000
13	[12|7222]	33 271	2 858 354	28 071 059	57 486 502	119 411 880
14	[12|6322]	15 023	1 218 340	11 938 339	24 497 442	50 840 856
15	[12|5422]	6 925	524 865	5 127 529	10 549 432	21 868 080
16	[12|5332]	6 354	478 000	4 668 838	9 607 816	19 914 210
17	[12|4432]	3 791	270 355	2 633 631	5 432 084	11 247 648
18	[12|4333]	2 968	206 029	2 004 538	4 139 156	8 566 236
19	[13|62222]	18 446	1 523 019	14 934 024	30 625 492	63 576 480
20	[13|53222]	7 455	569 930	5 570 699	11 456 502	23 752 680
21	[13|44222]	4 243	306 503	2 987 531	6 158 764	12 755 328
22	[13|43322]	3 213	225 145	2 191 561	4 523 464	9 363 312
23	[13|33332]	2 077	138 016	1 339 771	2 771 984	5 731 722
24	[14|522222]	8 728	677 955	6 631 182	13 629 060	28 264 800
25	[14|432222]	3 429	242 207	2 358 665	4 866 588	10 075 200
26	[14|333222]	2 023	133 966	1 300 243	2 690 606	5 563 080
27	[15|4222222]	3 599	255 799	2 492 015	5 140 108	10 642 944
28	[15|3322222]	1 868	122 419	1 187 578	2 458 596	5 082 336
29	[16|32222222]	1 583	101 431	982 863	2 036 876	4 208 640
30	[17|222222222]	1 135	69 175	668 431	1 388 684	2 866 176

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