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ON ANTI-PERIODIC SOLUTIONS FOR FUZZY BAM NEURAL NETWORKS WITH CONSTANT DELAYS ON TIME SCALES

HONGMEI BAO

Faculty of Mathematics and Physics
Huaiyin Institute of Technology
Huai'an, Jiangsu, 223003, P.R. CHINA

ABSTRACT: By applying analysis method on time scales and constructing suitable Lyapunov functional, some sufficient conditions are established for the existence and global exponential stability of anti-periodic solutions for a kind of fuzzy BAM neural networks on time scales. Moreover an example is given to illustrate our results.

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1. INTRODUCTION

Bi-directional associative memory (BAM) neural network model, proposed by Kosko [1, 2], is a two-layer nonlinear feedback network model. It is often known as an extension of the unidirectional auto-associator of the Hopfield model, generalizing the single-layer auto-associative Hebbian correlation to a two-layer pattern- matched hetero-associative circuit. It has promising potential for applications in many different fields such as associative memory, artificial intelligence, and some optimization problems. In such applications, it is of prime importance to ensure that the designed neural networks are stable. In recent years, the dynamical behaviors on the existence and global stability of equilibrium, periodic solutions of BAM neural networks with constant delays or time-varying delays or distributed delays have been studied by some scholars (see for example Refs. [3, 4, 5, 6, 7, 8, 9, 10, 11, 12] and the references therein). Also, there are some papers to study the dynamics of the discrete time neural networks, such as Refs. [13, 14, 15]. However, most of the investigations focused on the continuous or discrete systems, respectively.

It is troublesome to study the dynamical properties for continuous and discrete systems, respectively. So it is significant to study dynamical systems on time scales which can unify continuous and discrete time models very well. The theory of time scales was initiated by S. Hilger (1988), it has a tremendous potential for applications in some mathematical models of real processes and phenomena studied in physics, population dynamics, biotechnology, economics and so on. The readers can refer the books by Bohner and Peterson [16, 17], which summarize much of time scales calculus. Also many dynamical results of neural networks on time scales have been obtained by many scholars (see [18, 19, 20] and references cited therein).

In this paper, we would like to integrate fuzzy operations into BAM neural networks. Speaking of fuzzy operations, Yang and Yang [21] first introduced fuzzy cellular neural networks (FCNNs) combining those operations with cellular neural networks. So far researchers have founded that FCNNs are useful in image processing, and some results have been reported on stability and periodicity of FCNNs [22, 23, 24, 25, 26, 27].

Arising from problems in applied sciences, the existence of anti-periodic solutions plays a key role in characterizing the behavior of nonlinear differential equations (see [28, 29, 30, 31, 32]). It is worth continuing the investigation of the existence and stability of anti-periodic solutions of fuzzy BAM neural networks. To the best of our knowledge, there are few published papers considering the anti-periodic solutions of fuzzy BAM neural networks.

Motivated by the above discussions, we consider the following fuzzy BAM neural networks with constant delays on time scales.

$$\begin{cases} x_i^{\Delta}(t) & = & -a_i(t)h_i(x_i(t)) + \sum_{j=1}^{m} c_{ji}(t)f_j(y_j(t-\tau_{ji})) \\ & & + \bigwedge_{j=1}^{m} \alpha_{ji}(t)f_j(y_j(t-\tau_{ji})) + I_i(t) \\ & & + \bigvee_{j=1}^{m} \beta_{ji}(t)f_j(y_j(t-\tau_{ji})), i = 1, 2, \cdots, n, \\ y_j^{\Delta}(t) & = & -b_j(t)\varrho_j(y_j(t)) + \sum_{i=1}^{n} d_{ij}(t)g_i(x_i(t-\delta_{ij})) \\ & & + \bigwedge_{i=1}^{n} p_{ij}(t)g_i(x_i(t-\delta_{ij})) + J_j(t) \\ & & + \bigvee_{i=1}^{n} q_{ij}(t)g_i(x_i(t-\delta_{ij})), j = 1, 2, \cdots, m. \end{cases} \tag{1}$$

where $t \in \mathbb{T}$, $\mathbb{T}$ is a periodic time scale which has the subspace topology inherited from the standard topology on $\mathbb{R}$. n and m correspond to the number of units in X-

layer and Y-layer, respectively. $x_i(t), y_j(t)$ are the states of the ith neuron in X-layer and jth neuron in Y layer; $a_i(t) > 0, b_j(t) > 0$ represent an amplification function of the ith neuron in X-layer and jth neuron in Y layer at time t. $c_{ji}(t), d_{ij}(t)$ represents the elements of the feedback template. $\alpha_{ji}(t), \beta_{ji}(t), p_{ij}(t), q_{ij}(t)$ are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively; τ_{ji}, δ_{ij} denote the axonal signal transmission delays; $f_j(\cdot), g_i(\cdot)$ are signal transmission functions. $I_i(t), J_j(t)$ are external input to the ith unit in X-layer and the jth unit in Y-layer,respectively.

Without loss of generality, we set $[a, b]_{\mathbb{T}} = \{t \in \mathbb{T} : a \le t \le b\}$ and $0 \in \mathbb{T}, \mathbb{T}$ is unbounded above, i.e. $\sup \mathbb{T} = \infty$. Let $x = (x_1, x_2, \cdots, x_k)^T \in \mathbb{R}^k$ denote a column vector, in which the symbol $(\cdot)^T$ represents the transpose of a vector. Let $|x|$ be the absolute-value vector given by $|x| = (|x_1|, |x_2|, \cdots, |x_k|)$, and $\|x\| = \sum_{i=1}^{k} |x_i|$.

Let $u(t) = (x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T \in C(\mathbb{T}, \mathbb{R}^{n+m})$, $u_i(t)$ is said to be an ω anti periodic on $\mathbb{T}$ if $x_i(t+\omega) = -x_i(t), y_j(t+\omega) = -y_j(t)$ for all $t \in \mathbb{T}, t+\omega \in \mathbb{T}$. The initial conditions associated with system (1) are of the form

$$\begin{cases} x_i(s) = \varphi_i(s), \quad s \in [-\tau, 0]_{\mathbb{T}}, \quad \tau = \max_{1\le i\le n, 1\le j\le m}\{\tau_{ji}\}, \\ \\ y_j(s) = \phi_j(s), \quad s \in [-\delta, 0]_{\mathbb{T}}, \quad \delta = \max_{1\le i\le n, 1\le j\le m}\{\delta_{ij}\}, \end{cases} \tag{2}$$

where $\varphi_i \in C([-\tau, 0], \mathbb{R}), \phi_j \in C([-\delta, 0], \mathbb{R}), i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.

For the sake of convenience, we introduce some notations

$$\overline{I}_i = \sup_{t\in\mathbb{T}} |I_i(t)|, \quad \overline{I} = \max_{1\le i\le n}\{\overline{I}_i\}, \overline{J}_j = \sup_{t\in\mathbb{T}} |J_j(t)|, \quad \overline{J} = \max_{1\le j\le m}\{\overline{J}_j\},$$

$$\overline{\alpha}_{ji} = \sup_{t\in\mathbb{T}} |\alpha_{ji}(t)|, \ \overline{\beta}_{ji} = \sup_{t\in\mathbb{T}} |\beta_{ji}(t)|, \quad \overline{p}_{ij} = \sup_{t\in\mathbb{T}} |p_{ij}(t)|. \ \overline{q}_{ij} = \sup_{t\in\mathbb{T}} |q_{ij}(t)|,$$

$$\overline{c}_{ji} = \sup_{t\in\mathbb{T}} |c_{ji}(t)|, \quad \overline{d}_{ij} = \sup_{t\in\mathbb{T}} |d_{ij}(t)|, \underline{a}_i = \inf_{t\in\mathbb{T}} |a_i(t)|, \quad \underline{b}_j = \inf_{t\in\mathbb{T}} |b_j(t)|.$$

Denote $\mathbb{R}^+ = (0, \infty), \mathbb{T}^+ = (0, \infty)_{\mathbb{T}}$, Throughout this paper, we make the following assumptions.

(**A1**) $a_i(t+\omega)h_i(r) = -a_i(t)h_i(-r)$, $b_j(t+\omega)\varrho_j(r) = -b_j(t)\varrho_j(-r)$, $c_{ji}(t+\omega)f_j(r) = -c_{ji}(t)f_j(-r)$, $\alpha_{ji}(t+\omega)f_j(r) = -\alpha_{ji}(t)f_j(-r)$, $\beta_{ji}(t+\omega)f_j(r) = -\beta_{ji}(t)f_j(-r)$, $d_{ij}(t+\omega)g_i(r) = -d_{ij}(t)g_i(-r)$, $p_{ij}(t+\omega)g_i(r) = -p_{ij}(t)g_i(-r)$, $q_{ij}(t+\omega)g_i(r) = -q_{ij}(t)g_i(-r)$, $I_i(t+\omega) = -I_i(t)$, $J_j(t+\omega) = -J_j(t)$ for all $t \in \mathbb{T}, r \in \mathbb{R}$.

(**A2**) $h_i, \varrho_j \in C(\mathbb{R}, \mathbb{R})$, there exist constants $\xi_i > 0, \kappa_j > 0$ such that

$$\xi_i|r_1 - r_2| \le \text{sgn}(r_1 - r_2)[h_i(r_1) - h_i(r_2)],$$

$$\kappa_j|r_1 - r_2| \le \text{sgn}(r_1 - r_2)[\varrho_j(r_1) - \varrho_j(r_2)],$$

for all $r_1, r_2 \in \mathbb{R}$ and $h_i(0) = 0, \varrho_j(0) = 0, i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.
(**A3**) $f_j, g_j \in C(\mathbb{R}, \mathbb{R})$, and there exist $l_j > 0, \nu_j > 0$ such that

$$|f_j(r_1) - f_j(r_2)| \leq l_j |r_1 - r_2|, \quad |g_i(r_1) - g_i(r_2)| \leq \nu_i |r_1 - r_2|,$$

for all $r_1, r_2 \in \mathbb{R}$ and $f_j = 0, g_i(0) = 0, i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.

The organization of the rest of this paper is as follows. In Section 2, we introduce some definitions and lemmas to make preparations for later sections. In Section 3, by using analysis method and constructing Lyapunov functional, we establish sufficient conditions for the existence of the anti-periodic solutions of system (1) which is globally exponentially stability. An example is given to demonstrate the effectiveness of our results in Section 4.

2. PRELIMINARIES

In this section, we shall first recall some basic definitions, lemmas which are used in what follows.

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+$ are defined, respectively, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \ \rho(t) = \sup\{s \in \mathbb{T} : s < t\}, \ \mu(t) = \sigma(t) - t.$$

A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T} \setminus \{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k = \mathbb{T} \setminus \{m\}$. otherwise $\mathbb{T}_k = \mathbb{T}$.

A function $f : \mathbb{T} \to R$ is right-dense continuous provided it is continuous at right-dense point in $\mathbb{T}$ and its left-side limits exist at left-dense points in $\mathbb{T}$. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{rd}(\mathbb{T}, \mathbb{R})$. If f is continuous at each right-dense point and each left-dense point, then f is said to be a continuous function on $\mathbb{T}$. We define $C(J, \mathbb{R}) = \{u(t) : u(t) \text{ is continuous on } J\}$.

Definition 1. [33] If $a \in \mathbb{T}, \sup \mathbb{T} = \infty$, and f is rd-continuous on $[0, \infty)_{\mathbb{T}}$, then we define the improper integral by

$$\int_a^\infty f(s)\Delta s := \lim_{b \to \infty} \int_a^b f(s)\Delta s,$$

provided this limit exists, and we say that the improper integral converges in this case. If this limit does not exist, then we say that the improper integral diverges.

Definition 2. [33] For each $t \in \mathbb{T}$, let N be a neighborhood of t, then, for $V \in C_{rd}[\mathbb{T} \times \mathbb{R}^n, \mathbb{R}^+)$. Define $D^+V^\Delta(t, x(t))$ to mean that, given $\varepsilon > 0$, there exists a right neighborhood $N_\varepsilon \subset N$ of t such that

$$\frac{V(\sigma(t), x(\sigma(t))) - V(s, x(\sigma(t))) - \mu(t,s)f(t, x(t))}{\mu(t,s)} < D^+V^\Delta(t, x(t)) + \varepsilon.$$

for each $s \in N_\varepsilon, s > t$, where $\mu(t,s) = \sigma(t) - s$. If t is rd and $V(t, x(t))$ is continuous at t, this reduces to

$$D^+V^\Delta(t, x(t)) = \frac{V(\sigma(t), x(\sigma(t))) - V(t, x(\sigma(t)))}{\sigma(t) - t}.$$

Definition 3. [16] We say that a time scale $\mathbb{T}$ is periodic if there exists $p > 0$ such that if $t \in \mathbb{T}$, then $t \pm p \in \mathbb{T}$. For $\mathbb{T} \neq \mathbb{R}$, the least positive p is called the period of the time scale.

Definition 4. [16] Let $\mathbb{T} \neq \mathbb{R}$ be a periodic time scale with periodic p. We say that the function $f : \mathbb{T} \to \mathbb{R}$ is ω anti-periodic if there exists a natural number n such that $\omega = np, f(t+\omega) = -f(t)$ for all $t \in \mathbb{T}$ and ω is the least number such that $f(t+\omega) = -f(t)$. If $\mathbb{T} = \mathbb{R}$, we say that f is ω anti-periodic if ω is the least positive number such that $f(t+\omega) = -f(t)$ for all $t \in \mathbb{T}$.

Definition 5. For $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$, we define the delta derivative of $y(t)$, $f^\Delta(t)$, to be the number (if exists) with the property that for given $\varepsilon > 0$, there exists a neighborhood U of t such that

$$|[f(\sigma(t)) - f(s)] - y^\Delta(t)[\sigma(t) - f(s)]| < \varepsilon|\sigma(t) - s|,$$

for all $s \in U$. If f is continuous, then f is right-dense continuous, and y is delta differentiable at t, then f is continuous at t. Let f be right-dense continuous. If $F^\Delta(t) = f(t)$, then we define the delta integral by $\int_a^t f(s)\Delta s = F(t) - F(a)$.

A function $r : \mathbb{T} \to R$ is called regressive if $1 + \mu(t)r(t) \neq 0$, for all $t \in \mathbb{T}^k$.

If r is regressive function, then the generalized exponential function e_r is defined by

$$e_r(t,s) = \exp\left\{\int_s^t \xi_{\mu(\tau)}(r(\tau))\Delta\tau\right\}, \quad s, t \in \mathbb{T},$$

with the cylinder transformation

$$\xi_{h(z)} = \begin{cases} \frac{\log(1+hz)}{h}, & h \neq 0 \\ z, & h = 0 \end{cases}$$

Let $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions, we define

$$p \oplus q := p + q + \mu pq; \ p \ominus q := p \oplus (\ominus q); \ \ominus p := \frac{p}{1 + \mu p}.$$

Lemma 1. *[17] Let p, q be regressive functions on $\mathbb{T}$. Then*
(i) $e_0(t,s) = 1$ and $e_p(t,t) = 1$; (ii) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t,s)$;
(iii) $e_p(t,s)e_p(s,r) = e_p(t,r)$; (iv) $e_p^{\Delta}(\cdot, s) = pe_p(\cdot, s)$.

Lemma 2. *[21] Suppose x and y are two states of system (1), then we have*

$$\left|\bigwedge_{j=1}^{n} \alpha_{ij}(t)g_j(x) - \bigwedge_{j=1}^{n} \alpha_{ij}(t)g_j(y)\right| \leq \sum_{j=1}^{n} |\alpha_{ij}(t)||g_j(x) - g_j(y)|,$$

and

$$\left|\bigvee_{j=1}^{n} \beta_{ij}(t)g_j(x) - \bigvee_{j=1}^{n} \beta_{ij}(t)g_j(y)\right| \leq \sum_{j=1}^{n} |\beta_{ij}(t)||g_j(x) - g_j(y)|.$$

Definition 6. Let $u^*(t) = (x_1^*(t), \cdots, x_n^*(t), y_1^*(t), \cdots, y_m^*(t))^T$ be the solution of system (1) with initial value $\theta^* = (\varphi_1^*(t), \cdots, \varphi_n^*(t), \phi_1^*(t), \cdots, \phi_m^*(t))^T$, it is said to be globally exponentially stable if for all solution of (1.1) $u(t) = (x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T$ with initial value $\theta = (\varphi_1(t), \cdots, \varphi_n(t), \phi_1(t), \cdots, \phi_m(t))^T$, there exists positive constant $\varepsilon > 0$ and $M = M(\varepsilon) \geq 1$ such that, for every $\eta \in \mathbb{T}$,

$$\sum_{i=1}^{n} |x_i(t) - x_i^*(t)| + \sum_{j=1}^{m} |y_j(t) - y_j^*(t)| \leq Me_{\Theta\varepsilon}(t,\eta)\|\theta - \theta^*\|.$$

where

$$\|\theta - \theta^*\| = \sum_{i=1}^{n} \sup_{\eta \in (-\delta, 0]_{\mathbb{T}}} |\varphi_i(\eta) - \varphi_i^*(\eta)| + \sum_{j=1}^{m} \sup_{\eta \in (-\tau, 0]_{\mathbb{T}}} |\phi_j(\eta) - \phi_i^*(\eta)|.$$

3. MAIN RESULT

In this section, applying analysis method and constructing proper Lyapunov functional, we will prove the existence of anti periodic solutions of (1) which is global exponential stability.

Lemma 3. *Let $(A1)-(A3)$ hold, further suppose that the following assumption hold*
$(A4)$ There exists a positive constant $\gamma > 0$ such that

$$-\underline{a}_i\xi_i + \sum_{j=1}^{m}(\overline{c}_{ji} + \overline{\alpha}_{ji} + \overline{\beta}_{ji})l_j < -\gamma < 0,$$

$$-\underline{b}_j\kappa_j + \sum_{i=1}^{n}(\overline{d}_{ij} + \overline{p}_{ij} + \overline{q}_{ij})\nu_i < -\gamma < 0.$$

Suppose that $u(t) = (x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T$ *is a solution of system (1) with the initial condition*

$$\begin{cases} x_i(s) = \varphi_i(s), \ |\varphi_i(s)| < \frac{\overline{I}}{\gamma}, \ s \in [-\tau, 0]_{\mathbb{T}}, \\ \\ y_j(s) = \phi_j(s), \ |\phi_j(s)| < \frac{\overline{J}}{\gamma}, \ s \in [-\delta, 0]_{\mathbb{T}} \end{cases} \tag{3}$$

Then

$$|x_i(t)| < \frac{\overline{I}}{\gamma}, \quad |y_j(t)| < \frac{\overline{J}}{\gamma}, \quad t \in [0, \infty)_{\mathbb{T}}, \ i = 1, 2, \cdots, n, \ j = 1, 2, \cdots, m.$$

Proof. By way of contradiction, we assume that (3) does not hold, then there exist $i \in \{1, 2, \cdots, n\}$ or $j \in \{1, 2, \cdots, m\}$, and the first $t_0 > 0, t_0 \in \mathbb{T}$ such that

$$|x_i(t_0)| \geq \frac{\overline{I}}{\gamma}, \ \ |x_i(\rho(t_0))| \leq \frac{\overline{I}}{\gamma}, \ \ |x_i(t)| < \frac{\overline{I}}{\gamma}, \ \ t \in [-\tau, t_0)_{\mathbb{T}}, \tag{4}$$

or

$$|y_j(t_0)| \geq \frac{\overline{J}}{\gamma}, \ \ |y_i(\rho(t_0))| \leq \frac{\overline{J}}{\gamma}, \ \ |y_j(t)| < \frac{\overline{J}}{\gamma}, \ \ t \in [-\delta, t_0)_{\mathbb{T}}, \tag{5}$$

If (4) hold, calculating the Dini derivative of $|x_i(t_0)$, together with $(A1) - (A4)$, we have

$$\begin{aligned}
0 \quad \leq \quad & D^+(|x_i(t_0)|^{\Delta}) = \operatorname{sgn}(x_i(t_0)) \{-a_i(t_0)h_i(x_i(t_0)) \\
& + \sum_{j=1}^{m} c_{ji}(t_0) f_j(y_j(t_0 - \tau_{ji})) \\
& \left. + \bigwedge_{j=1}^{m} \alpha_{ji}(t_0) f_j(y_j(t_0 - \tau_{ji})) + \bigvee_{j=1}^{m} \beta_{ji}(t_0) f_j(y_j(t_0 - \tau_{ji})) + I_i(t_0) \right\} \\
\leq \quad & -a_i(t_0)|h_i(x_i(t_0)) - h_i(0)| + \sum_{j=1}^{m} |c_{ji}(t_0)||f_j(y_j(t_0 - \tau_{ji})) - f_j(0)| \\
& + \left| \bigwedge_{j=1}^{m} \alpha_{ji}(t_0) f_j(y_j(t_0 - \tau_{ji})) - \bigwedge_{j=1}^{m} \alpha_{ji}(t_0) f_j(0) \right| \\
& + \left| \bigvee_{j=1}^{m} \beta_{ji}(t_0) f_j(y_j(t_0 - \tau_{ji})) - \bigvee_{j=1}^{m} \beta_{ji}(t_0) f_j(0) \right| + |I_i(t_0)| \\
\leq \quad & -\underline{a}_i \xi_i |x_i(t_0)| + \sum_{j=1}^{m} (\overline{c}_{ji} + \overline{\alpha}_{ji} + \overline{\beta}_{ji}) l_j |y_j(t_0 - \tau_{ji})| + \overline{I}_i \\
\leq \quad & \left[-\underline{a}_i \xi_i + \sum_{j=1}^{m} (\overline{c}_{ji} + \overline{\alpha}_{ji} + \overline{\beta}_{ji}) l_j \right] \frac{\overline{I}}{\gamma} + \overline{I}_i \\
\leq \quad & 0,
\end{aligned}$$

which is a contradiction. Similarly, we can prove that (5) does not hold. The proof of Lemma 3 is completed. □

Theorem 7. *Assume (A1)-(A4) hold. Suppose further that*
(A5) there exist positive constants $\varepsilon > 0, \zeta_i > 0, \zeta'_j > 0$ such that

$$\zeta_i[\varepsilon - \underline{a}_i\xi_i(1+\mu(t)\varepsilon)] + \sum_{j=1}^{m}\zeta'_j(\overline{d}_{ij} + \overline{p}_{ij} + \overline{q}_{ij})\nu_i(1+\varepsilon\mu(t+\delta_{ij}))e_\varepsilon(t+\delta_{ij}, t) < 0,$$

$$\zeta'_j[\varepsilon - \underline{b}_j\kappa_j(1+\mu(t)\varepsilon)] + \sum_{i=1}^{n}\zeta_i(\overline{c}_{ji} + \overline{\alpha}_{ji} + \overline{\beta}_{ji})l_j(1+\varepsilon\mu(t+\tau_{ji}))e_\varepsilon(t+\tau_{ji}, t) < 0,$$

Then the solution of system (1) is globally exponentially stable.

Proof. Let $u^*(t) = (x_1^*(t), \cdots, x_n^*(t), y_1^*(t), \cdots, y_m^*(t))^T$ be the solution of (1.1) with initial value $\theta^* = (\varphi_1^*(t), \cdots, \varphi_n^*(t), \phi_1^*(t), \cdots, \phi_m^*(t))^T$, and $u^(t) = (x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T$ be the solution of (1) with initial value $\theta = (\varphi_1(t), \cdots, \varphi_n(t), \phi_1(t), \cdots, \phi_m(t))^T$. Then we have

$$\begin{aligned}(x_i(t) \quad - \quad & x_i^*(t))^\Delta \\ = \quad & -a_i(t)[h_i(x_i(t)) - h_i(x_i^*(t))] \\ + \quad & \sum_{j=1}^{m} c_{ji}(t)[f_j(y_j(t-\tau_{ji})) - f_j(y_j^*(t-\tau_{ji}))] \\ + \quad & \bigwedge_{j=1}^{m} \alpha_{ji}(t)f_j(y_j(t-\tau_{ji})) - \bigwedge_{j=1}^{m} \alpha_{ji}(t)f_j(y_j^*(t-\tau_{ji})) \\ + \quad & \bigvee_{j=1}^{m} \beta_{ji}(t)f_j(y_j(t-\tau_{ji})) - \bigvee_{j=1}^{m} \beta_{ji}(t)f_j(y_j^*(t-\tau_{ji}))\end{aligned} \tag{6}$$

and

$$\begin{aligned}(y_j(t) \quad - \quad & y_j^*(t))^\Delta \\ = \quad & -b_j(t)[\varrho_j(y_j(t)) - \varrho_j(y_j^*(t))] \\ + \quad & \sum_{i=1}^{n} d_{ij}(t)[g_i(x_i(t-\delta_{ij})) - g_i(x_i^*(t-\delta_{ij}))] \\ + \quad & \bigwedge_{i=1}^{n} p_{ij}(t)g_i(x_i(t-\delta_{ij})) - \bigwedge_{i=1}^{m} p_{ij}(t)g_i(x_i^*(t-\delta_{ij})) \\ + \quad & \bigvee_{i=1}^{n} q_{ij}(t)g_i(x_i(t-\delta_{ij})) - \bigvee_{i=1}^{n} q_{ij}(t)g_i(x_i^*(t-\delta_{ij}))\end{aligned} \tag{7}$$

In view of (6) and (7), for $t > 0$, we have

$$D^+|x_i(t) \quad - \quad x_i^*(t)|^\Delta$$

$$\begin{aligned} \leq\ & -\underline{a}_i\xi_i|x_i(t)-x_i^*(t)| \\ & +\sum_{j=1}^{m}(\overline{c}_{ji}+\overline{\alpha}_{ji}+\overline{\beta}_{ji})l_j|y_j(t-\tau_{ji})-y_j^*(t-\tau_{ji})| \end{aligned} \quad (8)$$

and

$$\begin{aligned} D^+|y_j(t) \ &-\ y_j^*(t)|^\Delta \\ &\leq\ -\underline{b}_j\kappa_j|y_j(t)-y_j^*(t)| \\ &+\ \sum_{i=1}^{n}(\overline{d}_{ij}+\overline{p}_{ij}+\overline{q}_{ij})\nu_i|x_j(t-\delta_{ij})-x_i^*(t-\delta_{ij})| \end{aligned} \quad (9)$$

For any $\eta\in[-\max\{\tau,\delta\},0]$, we construct the Lyapunov functional

$$V(t)=\sum_{k=1}^{4}V_k(t).$$

where

$$V_1(t)=\sum_{i=1}^{n}\zeta_i e_\varepsilon(t,\eta)|x_i(t)-x_i^*(t)|,$$

$$\begin{aligned} V_2(t) \ &=\ \sum_{i=1}^{n}\sum_{j=1}^{m}\zeta_i(\overline{c}_{ji}+\overline{\alpha}_{ji}+\overline{\beta}_{ji})l_j \\ &\quad\times\int_{t-\tau_{ji}}^{t}(1+\varepsilon\mu(s+\tau_{ji}))e_\varepsilon(s+\tau_{ji},\eta)|y_j(s)-y_j^*(s)|\Delta s \end{aligned}$$

$$V_3(t)=\sum_{j=1}^{m}\zeta_j' e_\varepsilon(t,\eta)|y_j(t)-y_j^*(t)|,$$

$$\begin{aligned} V_4(t) \ &=\ \sum_{j=1}^{m}\sum_{i=1}^{n}\zeta_j'(\overline{d}_{ij}+\overline{p}_{ij}+\overline{q}_{ij})\nu_i \\ &\quad\times\int_{t-\delta_{ij}}^{t}(1+\varepsilon\mu(s+\delta_{ij}))e_\varepsilon(s+\delta_{ij},\eta)|x_i(s)-x_i^*(s)|\Delta s \end{aligned}$$

Calculating the delta derivative $D^+V^\Delta(t)$ along to (1), we can get

$$\begin{aligned} & D^+V_1(t)^\Delta|_{(1)} \\ =\ & \sum_{i=1}^{n}\zeta_i\left[\varepsilon e_\varepsilon(t,\eta)|x_i(t)-x_i^*(t)|+e_\varepsilon(\sigma(t),\eta)D^+|x_i(t)-x_i^*(t)|^\Delta\right] \\ \leq\ & \sum_{i=1}^{n}\zeta_i\left\{\varepsilon e_\varepsilon(t,\eta)|x_i(t)-x_i^*(t)|+e_\varepsilon(\sigma(t),\eta)\left[-\underline{a}_i\xi_i|x_i(t)-x_i^*(t)|\right.\right. \end{aligned}$$

$$
\begin{aligned}
& \quad + \sum_{j=1}^{m} (\overline{c}_{ji} + \overline{\alpha}_{ji} + \overline{\beta}_{ji}) l_j |y_j(t-\tau_{ji}) - y_j^*(t-\tau_{ji})| \Bigg] \Bigg\} \\
\leq & \quad \left\{ \sum_{i=1}^{n} \zeta_i \left[\varepsilon - \underline{a}_i \xi_i (1+\mu(t)\varepsilon)\right] e_\varepsilon(t,\eta) |x_i(t) - x_i^*(t)| \right\} \\
& \quad + \left\{ (1+\mu(t)\varepsilon) e_\varepsilon(t,\eta) \sum_{i=1}^{n} \sum_{j=1}^{m} \zeta_i (\overline{c}_{ji} + \overline{\alpha}_{ji} + \overline{\beta}_{ji}) l_j \right. \\
& \quad \times |y_j(t-\tau_{ji}) - y_j^*(t-\tau_{ji})| \}
\end{aligned}
$$

$$
\begin{aligned}
D^+ V_2(t)^\Delta|_{(1)} \leq & \sum_{i=1}^{n} \sum_{j=1}^{m} \zeta_i (\overline{c}_{ji} + \overline{\alpha}_{ji} + \overline{\beta}_{ji}) l_j (1+\varepsilon\mu(t+\tau_{ji})) \\
& \times e_\varepsilon(t+\tau_{ji},\eta) |y_j(t) - y_j^*(t)| \\
& - \sum_{i=1}^{n} \sum_{j=1}^{m} \zeta_i (\overline{c}_{ji} + \overline{\alpha}_{ji} + \overline{\beta}_{ji}) l_j (1+\varepsilon\mu(t)) \\
& \times e_\varepsilon(t,\eta) |y_j(t-\tau_{ji}) - y_j^*(t-\tau_{ji})|
\end{aligned}
$$

$$
\begin{aligned}
& \quad D^+ V_3(t)^\Delta|_{(1)} \\
= & \quad \sum_{j=1}^{m} \zeta_j' \left[\varepsilon e_\varepsilon(t,\eta) |y_j(t) - y_j^*(t)| + e_\varepsilon(\sigma(t),\eta) D^+ |y_j(t) - y_j^*(t)|^\Delta \right] \\
\leq & \quad \sum_{j=1}^{m} \zeta_j' \left\{ \varepsilon e_\varepsilon(t,\eta) |y_j(t) - y_j^*(t)| + e_\varepsilon(\sigma(t),\eta) \left[-\underline{b}_j \kappa_j |y_j(t) - y_j^*(t)| \right. \right. \\
& \quad + \sum_{i=1}^{n} (\overline{d}_{ij} + \overline{p}_{ij} + \overline{q}_{ij}) \nu_i |x_i(t-\delta_{ij}) - x_i^*(t-\delta_{ij})| \Bigg] \Bigg\} \\
\leq & \quad \left\{ \sum_{j=1}^{m} \zeta_j' \left[\varepsilon - \underline{b}_j \kappa_j (1+\mu(t))\varepsilon\right] e_\varepsilon(t,\eta) |y_j(t) - y_j^*(t)| \right\} \\
& \quad + \left\{ (1+\mu(t)\varepsilon) e_\varepsilon(t,\eta) \sum_{j=1}^{m} \sum_{i=1}^{n} \zeta_j' (\overline{d}_{ij} + \overline{p}_{ij} + \overline{q}_{ij}) \nu_i \right. \\
& \quad \times |x_i(t-\delta_{ij}) - x_i^*(t-\delta_{ij})| \}
\end{aligned}
$$

$$
\begin{aligned}
D^+ V_4(t)^\Delta|_{(1)} \leq & \sum_{j=1}^{m} \sum_{i=1}^{n} \zeta_j' (\overline{d}_{ij} + \overline{p}_{ij} + \overline{q}_{ij}) \nu_i (1+\varepsilon\mu(t+\delta_{ij})) \\
& \times e_\varepsilon(t+\delta_{ij},\eta) |x_i(t) - x_i^*(t)| \\
& - \sum_{j=1}^{m} \sum_{i=1}^{n} \zeta_j' (\overline{d}_{ij} + \overline{p}_{ij} + \overline{q}_{ij}) \nu_i (1+\varepsilon\mu(t)) e_\varepsilon(t,\eta)
\end{aligned}
$$

$$\times|x_i(t-\delta_{ij})-x_i^*(t-\delta_{ij})|$$

From the above, it follows that

$$\begin{aligned}
& D^+V(t)^\Delta \\
\leq & \sum_{i=1}^{n}\{\zeta_i\left[\varepsilon-\underline{a}_i\xi_i(1+\varepsilon\mu(t))\right] \\
& +\sum_{j=1}^{m}\zeta_j'(\overline{d}_{ij}+\overline{p}_{ij}+\overline{q}_{ij})\nu_i(1+\varepsilon\mu(t+\delta_{ij}))\Bigg\}e_\varepsilon(t,\eta)|x_i(t)-x_i^*(t)| \\
& +\sum_{i=j}^{m}\Bigg\{\zeta_j'\left[\varepsilon-\underline{b}_j\kappa_j(1+\varepsilon\mu(t))\right]+\sum_{i=1}^{n}\zeta_i(\overline{c}_{ji}+\overline{\alpha}_{ji}+\overline{\beta}_{ji}) \\
& \times l_j(1+\varepsilon\mu(t+\tau_{ji}))\}\,e_\varepsilon(t,\eta)|y_j(t)-y_j^*(t)| \qquad (10)
\end{aligned}$$

By assumption (A5), we obtain $D^+V(t)^\Delta<0$, i.e. $V(t)<V(0)$, for $t>0$.

On the other hand, we have

$$\begin{aligned}
V(0) = & \sum_{i=1}^{n}\zeta_i e_\varepsilon(0,\eta)|x_i(0)-x_i^*(0)|+\sum_{j=1}^{m}\zeta_j' e_\varepsilon(0,\eta)|y_j(0)-y_j^*(0)| \\
& +\sum_{i=1}^{n}\sum_{j=1}^{m}\zeta_i(\overline{c}_{ji}+\overline{\alpha}_{ji}+\overline{\beta}_{ji})l_j \\
& \times\int_{-\tau_{ji}}^{0}(1+\varepsilon\mu(s+\tau_{ji}))e_\varepsilon(s+\tau_{ji},\eta)|y_j(s)-y_j^*(s)|\Delta s \\
& +\sum_{j=1}^{m}\sum_{i=1}^{n}\zeta_j'(\overline{d}_{ij}+\overline{p}_{ij}+\overline{q}_{ij})\nu_i \\
& \times\int_{-\delta_{ij}}^{0}(1+\varepsilon\mu(s+\delta_{ij}))e_\varepsilon(s+\delta_{ij},\eta)|x_i(s)-x_i^*(s)|\Delta s \\
\leq & \sum_{i=1}^{n}\Bigg\{\zeta_i e_\varepsilon(0,\eta)+\sum_{j=1}^{m}\zeta_j'(\overline{d}_{ij}+\overline{p}_{ij}+\overline{q}_{ij})\nu_i \\
& \times\int_{-\delta_{ij}}^{0}(1+\varepsilon\mu(s+\delta_{ij}))e_\varepsilon(s+\delta_{ij},\eta)\Delta s\Bigg\}\sup_{s\in[-\delta,0]}|x_i(s)-x_i^*(s)| \\
& +\sum_{j=1}^{m}\Bigg\{\zeta_j' e_\varepsilon(0,\eta)+\sum_{i=1}^{n}\zeta_i(\overline{c}_{ji}+\overline{\alpha}_{ji}+\overline{\beta}_{ji})l_j \\
& \times\int_{-\tau_{ji}}^{0}(1+\varepsilon\mu(s+\tau_{ji}))e_\varepsilon(s+\tau_{ji},\eta)\Bigg\}\sup_{s\in[-\tau,0]}|y_j(s)-y_j^*(s)| \\
\leq & \overline{M}(\varepsilon)\left\{\sup_{s\in[-\delta,0]}|x_i(s)-x_i^*(s)|+\sup_{s\in[-\tau,0]}|y_j(s)-y_j^*(s)|\right\}
\end{aligned}$$

where

$$\begin{aligned}\overline{M}(\varepsilon) &= \max\left\{\sum_{i=1}^{n}\left[\zeta_i e_\varepsilon(0,\eta)+\sum_{j=1}^{m}\zeta_j'(\overline{d}_{ij}+\overline{p}_{ij}+\overline{q}_{ij})\nu_i\right.\right.\\ &\quad\left.\times\int_{-\delta_{ij}}^{0}(1+\varepsilon\mu(s+\delta_{ij}))e_\varepsilon(s+\delta_{ij},\eta)\Delta s\right],\\ &\quad\sum_{j=1}^{m}\left[\zeta_j' e_\varepsilon(0,\eta)+\sum_{i=1}^{n}\zeta_i(\overline{c}_{ji}+\overline{\alpha}_{ji}+\overline{\beta}_{ji})l_j\right.\\ &\quad\left.\left.\times\int_{-\tau_{ji}}^{0}(1+\varepsilon\mu(s+\tau_{ji}))e_\varepsilon(s+\tau_{ji},\eta)\right]\right\}\end{aligned}$$

Also,

$$\sum_{i=1}^{n}\zeta_i e_\varepsilon(t,\eta)|x_i(t)-x_i^*(t)|+\sum_{j=1}^{m}\zeta_j' e_\varepsilon(t,\eta)|y_j(t)-y_j^*(t)|\le V(t)\le V(0),$$

that is

$$\begin{aligned}&\min_{1\le i\le n,1\le j\le m}\{\zeta_i,\zeta_j'\}e_\varepsilon(t,\eta)\left[\sum_{i=1}^{n}|x_i(t)-x_i^*(t)|+\sum_{j=1}^{m}|y_j(t)-y_j^*(t)|\right]\\ \le\ & V(0).\end{aligned}$$

We can obtain that

$$\sum_{i=1}^{n}|x_i(t)-x_i^*(t)|+\sum_{j=1}^{m}|y_j(t)-y_j^*(t)|\le M(\varepsilon)e_{\Theta\varepsilon}(t,\eta)\|\theta-\theta^*\|.$$

where $M(\varepsilon)=\frac{\overline{M}(\varepsilon)}{\min_{1\le i\le n,1\le j\le m}\{\zeta_i,\zeta_j'\}}$. Therefore, by Definition 6, the solution $u(t)$ of (1) is globally exponentially stable. This completes the proof. □

Theorem 8. *Assume that* $(A1)-(A5)$ *hold. Then system (1) has an* ω *anti periodic solution which is globally exponentially stable.*

Proof. Let $u(t)=(x_1(t),\cdots,x_n(t),y_1(t),\cdots,y_m(t))^T$ be a solution of (1) with initial conditions (3). Then by Lemma 3, the solution $u(t)$ is bounded.

It follows that

$$\begin{aligned}&((-1)^{k+1}x_i(t+(k+1)\omega))^\Delta=(-1)^{k+1}x_i^\Delta(t+(k+1)\omega)\\ =\ &(-1)^{k+1}\{-a_i(t+(k+1)\omega)h_i(t+(k+1)\omega)\\ &+\sum_{j=1}^{m}c_{ji}(t+(k+1)\omega)f_j(y_j(t+(k+1)\omega-\tau_{ji}))\end{aligned}$$

$$
\begin{aligned}
& + \bigwedge_{j=1}^{m} \alpha_{ji}(t+(k+1)\omega) f_j(y_j(t+(k+1)\omega - \tau_{ji})) \\
& + \bigvee_{j=1}^{m} \beta_{ji}(t+(k+1)\omega) f_j(y_j(t+(k+1)\omega - \tau_{ji})) + I_i(t+(k+1)\omega) \Bigg\} \\
= \; & -a_i(t) h_i((-1)^{k+1} x_i(t+(k+1)\omega) \\
& + \sum_{j=1}^{m} c_{ji}(t) f_j((-1)^{k+1} y_j(t+(k+1)\omega - \tau_{ji})) \\
& + \bigwedge_{j=1}^{m} \alpha_{ji}(t) f_j((-1)^{k+1} y_j(t+(k+1)\omega - \tau_{ji})) \\
& + \bigvee_{j=1}^{m} \beta_{ji}(t) f_j((-1)^{k+1} y_j(t+(k+1)\omega - \tau_{ji})) + I_i(t) \qquad (11)
\end{aligned}
$$

Similarly, we have

$$
\begin{aligned}
& ((-1)^{k+1} y_j(t+(k+1)\omega))^{\Delta} = (-1)^{k+1} y_j^{\Delta}(t+(k+1)\omega) \\
= \; & -b_j(t) \varrho_j((-1)^{k+1} y_j(t+(k+1)\omega) \\
& + \sum_{i=1}^{n} d_{ij}(t) g_i((-1)^{k+1} x_i(t+(k+1)\omega - \delta_{ij})) \\
& + \bigwedge_{i=1}^{n} p_{ij}(t) g_i((-1)^{k+1} x_i(t+(k+1)\omega - \delta_{ij})) \\
& + \bigvee_{i=1}^{n} q_{ij}(t) g_i((-1)^{k+1} x_i(t+(k+1)\omega - \delta_{ij})) + J_j(t) \qquad (12)
\end{aligned}
$$

Thus for any natural number k, $(-1)^{k+1} u_i(t+(k+1)\omega)$ is the solution of (1). By Theorem 7, there exists a constant $M(\varepsilon)$ such that

$$
\begin{aligned}
& \sum_{i=1}^{n} |(-1)^{k+1} x_i(t+(k+1)\omega) - (-1)^k x_i(t+k\omega)| \\
& + \sum_{j=1}^{m} |(-1)^{k+1} y_j(t+(k+1)\omega) - (-1)^k y_j(t+k\omega)| \\
\le \; & M(\varepsilon) e_{\Theta\varepsilon}(t+k\omega, \eta) \Bigg\{ \sum_{i=1}^{n} \sup_{s\in[-\tau,0]} |x_i(s+\omega) - x_i(s)| \\
& + \sum_{j=1}^{m} \sup_{s\in[-\delta,0]} |y_j(s+\omega) - y_j(s)| \Bigg\} \\
\le \; & M(\varepsilon) e_{\Theta\varepsilon}(t+k\omega, \eta) \frac{n\overline{I} + m\overline{J}}{\gamma} \qquad (13)
\end{aligned}
$$

Then for a natural number l, we have

$$\begin{aligned} & |(-1)^{l+1}x_i(t+(l+1)\omega)| \\ = & \left|x_i(t)+\sum_{k=0}^{l}\left[(-1)^{k+1}x_i(t+(k+1)\omega)-(-1)^k x_i(t+k\omega)\right]\right| \\ \leq & |x_i(t)| \\ & +\sum_{k=0}^{l}\left|(-1)^{k+1}x_i(t+(k+1)\omega)-(-1)^k x_i(t+k\omega)\right| \end{aligned} \tag{14}$$

$$\begin{aligned} & |(-1)^{l+1}y_j(t+(l+1)\omega)| \\ = & \left|y_j(t)+\sum_{k=0}^{l}\left[(-1)^{k+1}y_j(t+(k+1)\omega)-(-1)^k y_j(t+k\omega)\right]\right| \\ \leq & |y_j(t)| \\ & +\sum_{k=0}^{l}\left|(-1)^{k+1}y_j(t+(k+1)\omega)-(-1)^k y_j(t+k\omega)\right| \end{aligned} \tag{15}$$

Noting that $u(t)$ is bounded and (13), then there exist a sufficient large constant $K>0$ and a positive constant Γ such that

$$\left|(-1)^{k+1}x_i(t+(k+1)\omega)-(-1)^k x_i(t+k\omega)\right| \leq \Gamma(e^{-c\omega})^k, k>K, \tag{16}$$

and

$$\left|(-1)^{k+1}y_j(t+(k+1)\omega)-(-1)^k y_j(t+k\omega)\right| \leq \Gamma(e^{-c\omega})^k, k>K, \tag{17}$$

where $i=1,2,\cdots,n, j=1,2,\cdots,m$. It follows from (14)-(17) that $(-1)^k u(t+k\omega)$ uniformly converges to a continuous function $v(t)=(x_1^*(t),\cdots,x_n^*(t),y_1^*(t),\cdots,y_n^*(t))^T$ in time scales sense.

Now we will prove that $v(t)$ is an ω anti periodic solution of (1). First, we have

$$\begin{aligned} v(t+\omega) &= \lim_{k\to\infty}(-1)^k u_i(t+k\omega+\omega) \\ &= -\lim_{k\to\infty}(-1)^{k+1}u_i(t+(k+1)\omega) = -v(t). \end{aligned}$$

Next, we shall prove $v(t)$ is a solution of (1). (11) and (12) imply that $\{(-1)^{k+1}u_i(t+(k+1)\omega)$ uniformly converges to a continuous function in the sense time scales. Letting $k\to\infty$, we have

$$\begin{aligned} (x_i^*(t))^{\Delta} = & -a_i(t)h_i(x_i^*(t)) \\ & +\sum_{j=1}^{m}c_{ji}(t)f_j(y_j^*(t-\tau_{ji}))+\bigwedge_{j=1}^{m}\alpha_{ji}(t)f_j(y_j^*(t-\tau_{ji})) \\ & +\bigvee_{j=1}^{m}\beta_{ji}(t)f_j(y_j^*(t-\tau_{ji}))+I_i(t), i=1,2,\cdots,n, \end{aligned}$$

$$\begin{aligned}(y_j^*(t))^{\Delta} &= -b_j(t)\varrho_j(y_j^*(t)) \\ &+\sum_{i=1}^{n} d_{ij}(t)g_i(x_i^*(t-\delta_{ij}))+\bigwedge_{i=1}^{n} p_{ij}(t)g_i(x_i^*(t-\delta_{ij})) \\ &+\bigvee_{i=1}^{n} q_{ij}(t)g_i(x_i^*(t-\delta_{ij}))+J_j(t), j=1,2,\cdots,m.\end{aligned}$$

Therefore $v(t)$ is a solution of (1), applying Theorem 7, we can show that $v(t)$ is globally exponentially stable. This completes the proof. □

4. AN EXAMPLE

Example 1 Consider the following fuzzy BAM neural networks with delays on time scales.

$$\left\{\begin{aligned} x_i^{\Delta}(t) &= -a_i(t)h_i(x_i(t))+\textstyle\sum_{j=1}^{2} c_{ji}(t)f_j(y_j(t-\tau_{ji})) \\ &+\textstyle\bigwedge_{j=1}^{2}\alpha_{ji}(t)f_j(y_j(t-\tau_{ji})) \\ &+\textstyle\bigvee_{j=1}^{2}\beta_{ji}(t)f_j(y_j(t-\tau_{ij}))+I_i(t), \\ y_j^{\Delta}(t) &= -b_j(t)\varrho_j(y_j(t))+\textstyle\sum_{i=1}^{2} d_{ij}(t)g_i(x_i(t-\delta_{ij})) \\ &+\textstyle\bigwedge_{i=1}^{2} p_{ij}(t)g_i(x_i(t-\delta_{ij})) \\ &+\textstyle\bigvee_{i=1}^{2} q_{ij}(t)g_i(x_i(t-\delta_{ij}))+J_j(t),\end{aligned}\right. \tag{18}$$

where $t\in\mathbb{T}, i,j=1,2, \mathbb{T}=\bigcup_{k\in Z}[\frac{1}{4}k,\frac{1}{4}(k+1)]$ is a periodic time scale.

$$a_1(t)=2.04+0.2|\sin(4\pi t)|, a_2(t)=2.05+0.3|\sin(4\pi t)|,$$

$$b_1(t)=2.56+0.1|\cos(4\pi t)|, b_2(t)=2.58+0.4|\cos(4\pi t)|,$$

$$c_{11}(t)=0.07\sin(4\pi t), c_{12}(t)=0.09\cos(4\pi t), c_{21}(t)=0.08\cos(4\pi t),$$

$$c_{22}(t)=0.06\sin(4\pi t), \alpha_{11}(t)=0.12\cos(4\pi t), \alpha_{12}(t)=0.11\sin(4\pi t),$$

$$\alpha_{21}(t)=0.09\sin(4\pi t), \alpha_{22}(t)=0.06\cos(4\pi t), \beta_{11}(t)=0.04\sin(4\pi t),$$

$$\beta_{12}(t)=0.05\cos(4\pi t), \beta_{21}(t)=0.07\cos(4\pi t), \beta_{22}(t)=0.06\sin(4\pi t),$$

$$d_{11}(t)=0.09\sin(4\pi t), d_{12}(t)=0.07\cos(4\pi t), d_{21}(t)=0.06\cos(4\pi t),$$

$$d_{22}(t)=0.08\sin(4\pi t), p_{11}(t)=0.14\cos(4\pi t), p_{12}(t)=0.1\sin(4\pi t),$$

$$p_{21}(t)=0.08\sin(4\pi t), p_{22}(t)=0.09\cos(4\pi t), q_{11}(t)=0.06\sin(4\pi t),$$

$$q_{12}(t) = 0.09\cos(4\pi t), q_{21}(t) = 0.07\cos(4\pi t), q_{22}(t) = 0.12\sin(4\pi t),$$

$$I_1(t) = 0.5\sin(4\pi t), I_2(t) = 0.7\cos(4\pi t),$$

$$f_j(r) = \frac{1}{5}|\sin r|, g_i(r) = \frac{1}{5}|\sin r|(i,j=1,2).$$

$$J_1(t) = 0.4\sin(4\pi t), J_2(t) = 0.6\cos(4\pi t),$$

$$h_i(r) = \frac{1}{2}\sin r, \varrho_j(r) = \frac{1}{3}\sin r, i,j=1,2.$$

then, we have

$$\underline{a}_1 = 2.04, \underline{a}_2 = 2.05, \underline{b}_1 = 2.56, \underline{b}_2 = 2.58, \overline{c}_{11} = 0.07, \overline{c}_{22} = 0.06,$$

$$\overline{c}_{12} = 0.09, \overline{c}_{21} = 0.08, \overline{\alpha}_{11} = 0.12, \overline{\alpha}_{22} = 0.06, \overline{\alpha}_{12} = 0.11, \overline{\alpha}_{21} = 0.09,$$

$$\overline{\beta}_{11} = 0.04, \overline{\beta}_{22} = 0.06, \overline{\beta}_{12} = 0.05, \overline{\beta}_{21} = 0.07, \overline{d}_{11} = 0.09, \overline{d}_{22} = 0.08,$$

$$\overline{d}_{12} = 0.07, \overline{d}_{21} = 0.06, \overline{p}_{11} = 0.14, \overline{p}_{22} = 0.09, \overline{p}_{12} = 0.1, \overline{p}_{21} = 0.08,$$

$$\overline{q}_{11} = 0.06, \overline{q}_{22} = 0.12, \overline{q}_{12} = 0.09, \overline{q}_{21} = 0.07, \overline{I}_1 = 0.5, \overline{I}_2 = 0.7,$$

$$\overline{J}_1 = 0.4, \overline{J}_2 = 0.6, l_j = \nu_i = \frac{1}{5}, \xi_i = \frac{1}{2}, \kappa_j = \frac{1}{3},$$

$$\tau_{ji} = 0.3, \delta_{ij} = 0.4, i,j=1,2.$$

It is easy to get

$$-\underline{a}_1\xi_1 + \sum_{j=1}^{2}(\overline{c}_{j1} + \overline{\alpha}_{j1} + \overline{\beta}_{j1})l_j = -0.926 < -0.6 < 0,$$

$$-\underline{a}_2\xi_2 + \sum_{j=1}^{2}(\overline{c}_{j2} + \overline{\alpha}_{j2} + \overline{\beta}_{j2})l_j = -0.939 < -0.6 < 0,$$

$$-\underline{b}_1\kappa_1 + \sum_{i=1}^{2}(\overline{d}_{i1} + \overline{p}_{i1} + \overline{q}_{i1})\nu_i = -0.753 < -0.6 < 0,$$

$$-\underline{b}_2\kappa_2 + \sum_{i=1}^{2}(\overline{d}_{i2} + \overline{p}_{i2} + \overline{q}_{i2})\nu_i = -0.75 < -0.6 < 0.$$

Let $\zeta_i = 1, \zeta'_j = 1, \varepsilon = 0.1, \mu(t) = \frac{1}{8}$, it is easy verify $(A5)$ hold.

Therefore we can see that conditions $(A1)-(A5)$ hold. By Theorem 3.1 and Theorem 8, system (18) has a $\frac{1}{4}$-anti-periodic solution which is globally exponentially stable (see fig.1).

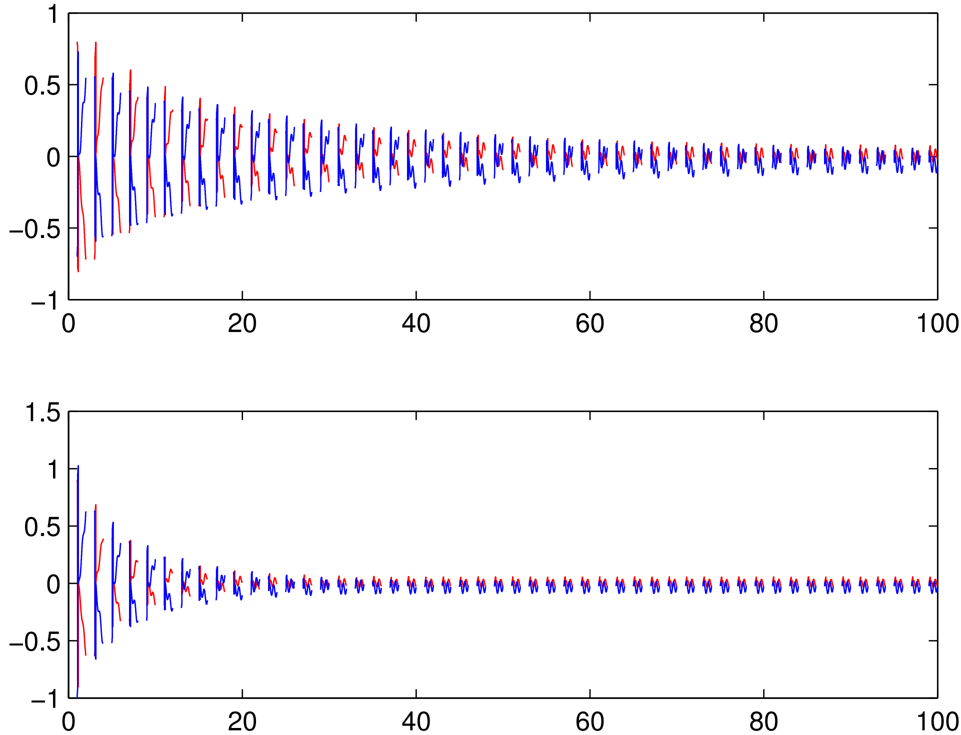

Figure 1: Numerical solution $x(t) = (x_1(t), x_2(t))^T, y(t) = (y_1(t), y_2(t))^T$ of systems (18) for initial value $\varphi(s) = (0.8, -0.7)^T$, $\phi(s) = (0.9, -1)^T$,$s \in [-0.4, 0]$.

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