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PRICING EQUITY-LINKED FOREIGN EXCHANGE OPTION UNDER A REGIME-SWITCHING MULTI-SCALE JUMP-DIFFUSION MODEL

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ABSTRACT: This paper studies the valuation of equity-linked foreign exchange call option under a regime-switching multi-scale jump diffusion model. The foreign equity price is driven by a regime-switching multi-scale jump-diffusion process and the foreign exchange rate is assumed to follow a regime-switching mean-reversion multi-scale jump-diffusion process. In addition, the correlations of the two processes are not only manifested in the diffusion parts but also in the jump components. The measure change and Fourier transform technique are adopted to calculate the price of equity-linked foreign exchange call option. Numerical examples and comparative analysis are also provided by fast Fourier transform algorithm to illustrate our results.

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1. INTRODUCTION

With the increasing fluctuations of exchange rate in the international markets and the development of international trade, more and more traders are seeking to avoid foreign exchange risk in a more effective way. The emergence of foreign exchange options can effectively avoid exchange rate risk, which are quite favored by the international financial markets. Foreign exchange options are contingent claims whose payoffs are determined in foreign currencies but are paid in their own currency, so the option holders face both foreign equity price volatility risk and exchange rate risk. There are different types of foreign exchange options depending on the structures of payoff.

Foreign exchange option pricing must take into account both the dynamic processes of foreign equity prices and exchange rates. Early studies (see [1],[2]) usually model the dynamics of underlying assets under the traditional Black-Scholes framework. Kwok and Wong [3] further provide the pricing of foreign exchange options with path-dependent characteristics. However, since the normality and continuity assumptions do not meet the needs of financial markets, many scholars have improved the models. Huang and Hung [4] give the pricing of foreign exchange options assuming that underlying price processes are correlated and driven by multi-dimensional Lévy processes. Xu et al. [5] consider the stochastic volatility with simultaneous jumps in prices and volatility based on [4] which can capture the foreign equity option prices more accurately. Xu et al. [6] extend the work of [3] and focus on the effect of higher order moments of asset prices on foreign exchange option prices.

One common feature of the aforementioned works is that the valuation of foreign equity option is studied under the Black-Scholes model, jump diffusion models and stochastic volatility model. In these traditional financial models, the market parameters are assumed to be independent of macroeconomic conditions. However, there are many empirical evidences that in a long span of time, the market behavior is significantly affected by economic factors. Regime-switching models which were first introduced into economics and finance by Hamilton [7] have provided us with a natural and convenient way to describe structural changes in market interest rate, exchange rate, stock returns, etc. In regime switching models, a continuous time, finite-state, Markov-chain is used to describe the different states of an economy. Over the past decade or two, there have been dozens of works on studying option valuation problems using regime-switching models (see [8], [9], [10], [11], [12], [13], etc.).

Frömmel et al. [14] introduce Markov regime switching into a monetary exchange rate model and provide a much better description of the data. In addition, Fan et al. [15] also incorporate the regime switching into mean-reversion lognormal model which describes the dynamic of foreign exchange rate and investigate the valuation of two foreign equity options with strike prices in the foreign currency (FEO_F) and in the domestic currency (FEO_D). However, they do not consider the impact of jumps which are due to the brusque variations caused by some rare events in currency exchange rate dynamics. For jump diffusion models, Martin [16], Tian et al. [17] and Niu and Wang [18] divide the jumps into individual jumps for each asset price respectively and common jumps that affect the prices of all assets. Inspired by these researches, to better describe both the time-inhomogeneity and sudden shocks in the processes of the foreign equity price and exchange rate, we propose regime-switching, mean-reversion and jumps which contain individual jumps and common jumps to model the foreign equity price and exchange rate in this paper. We named this model a regime-switching multi-scale jump-diffusion model. The correlations of the two processes are not only manifested in the diffusion parts but also in the jump parts. Our model combines the advantages of regime-switching models, mean-reversion models and multi-scale jump-diffusion models. We extend the model of [15] and consider the pricing of equity-linked foreign exchange call option.

In order to facilitate the calculation, the dimension of the problem is firstly reduced by introducing a new measure, and the characteristic function of the logarithmic exchange rate under the new measure is deduced. Furthermore, due to the complexity of regime-switching, the jump process involved, we employ the Fourier transform method to obtain the closed-form formula for equity-linked foreign exchange call option prices. Finally, we provide numerical results with log-double-exponential jump amplitude to describe all the jumps by using the fast Fourier transform algorithm proposed by Carr and Madan [19].

The rest of the paper is organized as follows. Section 2 presents the basic stochastic model of the foreign equity and the exchange rate. In section 3, the equity-linked foreign exchange option pricing formula is obtained by the Fourier transformation method. In section 4, a numerical analysis is performed to discuss the effects of various factors on the equity-linked foreign exchange option prices. Section 5 concludes the paper.

2. THE MODEL DESCRIPTION

Consider a continuous-time financial market with a finite time horizon $\mathcal{T} := [0, T]$, where $T < \infty$. We consider a complete probability space $(\Omega, \mathcal{F}, Q)$, where Q is a risk-neutral probability measure, under which all stochastic processes are defined. We equip the probability space $(\Omega, \mathcal{F}, Q)$ with a filtration $\mathcal{F} := \{\mathcal{F}_t \mid t \in \mathcal{T}\}$. $U = \{U_t\}_{t\in\mathcal{T}}$ is a continuous-time finite-state observable Markov chain on $(\Omega, \mathcal{F}, Q)$ with a finite state space $\mathcal{S} := (s_1, s_2, \cdots, s_N)$. We use the states of U to model the states of the economy. We adopt the assumptions of [8] that the state space of U is a limited collection of vectors $\{e_1, e_2, \cdots, e_N\}$, where $e_i = (0, \cdots, 1, \cdots, 0) \in \mathbf{R}^N$ with "1" in the ith component. Suppose the time-invariant matrix $\mathbf{A}$ denotes the generator or Q-matrix $(a_{ij})_{i,j=1,2,\cdots,N}$ of U, where a_{ij} is an infinitesimal intensity of U with $a_{ii} = -\sum_{i\neq j,j=1}^{N} a_{ij}$. Then, following [8], the semi-martingale decomposition of U is given by

$$dU_t = \mathbf{A}U_t dt + dM_t, \tag{1}$$

where $M = \{M_t\}_{t\in\mathcal{T}}$ is an $\mathbf{R}^N$-valued martingale with respect to the filtration generated by $\{U_t\}_{t\in\mathcal{T}}$ under Q.

We assume that S_t is the foreign equity price and F_t is the exchange rate in the domestic/foreign currency. Let the price processes of the foreign equity S_t and the exchange rate F_t follow regime-switching multi-scale jump-diffusion processes under the risk-neutral measure Q,

$$\frac{dS_t}{S_{t-}} = (\theta_t - k_1\lambda_1 - k_3\lambda_3)dt + \sigma_t dW_t^{(1)} + \left(e^{Z_{t-}^{(1)}} - 1\right) dN_t^{(1)} + \left(e^{Z_{t-}^{(3)}} - 1\right) dN_t^{(3)}, \tag{2}$$

$$\frac{dF_t}{F_{t-}} = (\alpha_t - \beta \ln F_t)dt + \gamma_t dW_t^{(2)} + \left(e^{Z_{t-}^{(2)}} - 1\right) dN_t^{(2)} + \left(e^{Z_{t-}^{(3)}} - 1\right) dN_t^{(3)}, \tag{3}$$

where $W_t^{(1)}$, $W_t^{(2)}$ are standard correlated Brownian motions on $(\Omega, \mathcal{F}, Q)$ and the instantaneous correlation coefficient at time t is given by:

$$< W_t^{(1)}, W_t^{(2)} >= \int_0^t \rho_s ds.$$

Such as Wong and Zhao [20], as long as currency option is concerned, the domestic interest rate and the foreign interest rate are embedded into the risk-neutral parameters θ_t and α_t. We assume that risk-neutral parameters θ_t and α_t, the volatility of the foreign equity σ_t, the volatility of the exchange rate γ_t and the instantaneous correlation coefficient ρ_t all depend on $\{U_t\}_{t\in\mathcal{T}}$. They are defined as

$$\theta_t =< \theta, U_t >, \sigma_t =< \sigma, U_t >, \alpha_t =< \alpha, U_t >, \gamma_t =< \gamma, U_t >, \rho_t =< \rho, U_t >,$$

where $\theta = (\theta_1, \theta_2, \cdots, \theta_N) \in \mathbf{R}^N$, $\sigma = (\sigma_1, \sigma_2, \cdots, \sigma_N) \in \mathbf{R}^N$, $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_N) \in \mathbf{R}^N$, $\gamma = (\gamma_1, \gamma_2, \cdots, \gamma_N) \in \mathbf{R}^N$ and $\rho = (\rho_1, \rho_2, \cdots, \rho_N) \in \mathbf{R}^N$ with $\sigma_i > 0$, $\gamma_i > 0$ for each $i = 1, 2, \cdots, N$. $< .,. >$ denotes the inner product in $\mathbf{R}^N$, for $i \neq j$, $< e_i, e_j >= 0$, else, $< e_i, e_i >= 1$. The parameter β, controlling the speed of mean reversion for the logarithmic foreign exchange rate process, is assumed to be a positive constant.

On the basis of [17], it is assumed that shocks to foreign equity price S_t and exchange rate F_t also contain two parts: individual shocks $N_t^{(1)}$, $N_t^{(2)}$ for each asset price respectively and common shocks $N_t^{(3)}$ affecting the prices of all assets. Moreover, we assume that $N_t^{(1)}, N_t^{(2)}$ and $N_t^{(3)}$ are Poisson processes with intensities λ_1, λ_2 and

λ_3 correspondly. If the jump occurs at time t, the jump amplitudes of $N_t^{(1)}, N_t^{(2)}$ and $N_t^{(3)}$ are controlled by $Z_t^{(1)}$, $Z_t^{(2)}$ and $Z_t^{(3)}$. For any time $t \neq s$, $i = 1, 2, 3$, we assume that $Z_t^{(i)}$ and $Z_s^{(i)}$ are independently and identically distributed and $Z_t^{(i)} \sim f_{Z^{(i)}}(z)$. The mean percentage jump size of the price is given by

$$k_i = E(e^{Z_t^{(i)}} - 1), i = 1, 2, 3,$$

where E denotes an expectation under the risk-neutral measure Q. Moreover, we assume that $(W_t^{(1)}, W_t^{(2)})$, $N_t^{(1)}$, $N_t^{(2)}$, $N_t^{(3)}$, $Z_t^{(1)}$, $Z_t^{(2)}$ and $Z_t^{(3)}$ are mutually independent.

Let $\mathbb{F}^S := \{\mathcal{F}_t^S | t \in \mathcal{T}\}$, $\mathbb{F}^F := \{\mathcal{F}_t^F | t \in \mathcal{T}\}$ and $\mathbb{F}^U := \{\mathcal{F}_t^U | t \in \mathcal{T}\}$ be the right-continuous, Q-complete, natural filtrations generated by processes S, F and U, respectively. Furthermore, we define the enlarged filtration $\mathbb{G} := \{\mathcal{G}_t | t \in \mathcal{T}\}$ by the minimal σ-field containing $\mathcal{F}_t^S$, $\mathcal{F}_t^F$ and $\mathcal{F}_t^U$. That is,

$$\mathcal{G}_t := \mathcal{F}_t^S \vee \mathcal{F}_t^F \vee \mathcal{F}_t^U, t \in \mathcal{T}.$$

For each $t \in \mathcal{T}$, $\mathcal{G}_t$ represents publicly available market information up to time t.

3. EQUITY-LINKED FOREIGN EXCHANGE OPTION PRICING

The payoff function of the equity-linked foreign exchange call option is given by

$$C(T) = S_T(F_T - K_F)^+,$$

where K_F is the strike price of exchange rate, S_T is the price of the underlying asset which is denominated in foreign currency, the holder wishes to buy foreign assets at the lowest price in the exchange rate. Let $C(0, T, K_F)$ denote the value of the option in initial time. By no arbitrage pricing theory, the following pricing formula is standard:

$$C(0, T, K_F) = E\left[e^{-\int_0^T r_t dt} S_T(F_T - K_F)^+\right], \tag{4}$$

where r_t denotes the domestic interest rate and $r_t =< r, U_t >$, $r = (r_1, r_2, \cdots, r_N) \in \mathbf{R}^N$. For each $i = 1, 2, \cdots, N, r_i > 0$. Let $k_F = \ln(K_F)$ be the logarithmic strike price of the exchange rate. As in [19], the modified equity-linked foreign exchange call option price is defined by

$$c(0, T, k_F) = e^{a_F k_F} C(0, T, K_F), \tag{5}$$

where a_F is a predetermined positive constant such that $c(0, T, k_F)$ is square integrable in k_F over the entire real line. Then the Fourier transform of $c(0, T, k_F)$ is as follows:

$$\psi(0, T, u) = \int_{-\infty}^{\infty} e^{iuk_F} c(0, T, k_F) dk_F. \tag{6}$$

For our purpose, we introduce a probability measure Q_S equivalent to Q on $\mathcal{G}_T$:

$$\left.\frac{dQ_S}{dQ}\right|_{\mathcal{G}_T} = \frac{S_T}{E[S_T|\mathcal{F}_T^U]}. \tag{7}$$

A direct calculation to formula (2.2) gives

$$\begin{aligned} S_T = S_0 \exp\Big\{ \int_0^T (\theta_t &- \frac{1}{2}\sigma_t^2)dt + \int_0^T \sigma_t dW_t^{(1)} + \int_0^T Z_t^{(1)} dN_t^{(1)} \\ &- k_1\lambda_1 T + \int_0^T Z_t^{(3)} dN_t^{(3)} - k_3\lambda_3 T\Big\}. \end{aligned} \tag{8}$$

Moreover, it is easy to see that given $\mathcal{F}_T^U$, the mean of the conditional distribution of S_T is that

$$E[S_T|\mathcal{F}_T^U] = S_0 e^{\int_0^T \theta_t dt}. \tag{9}$$

Substituting (3.5) and (3.6) into (3.4) yields

$$\begin{aligned} \left.\frac{dQ_S}{dQ}\right|_{\mathcal{G}_T} = \exp\Big\{ \int_0^T \sigma_t dW_t^{(1)} &- \frac{1}{2}\int_0^T \sigma_t^2 dt + \int_0^T Z_t^{(1)} dN_t^{(1)} \\ &- k_1\lambda_1 T + \int_0^T Z_t^{(3)} dN_t^{(3)} - k_3\lambda_3 T\Big\}. \end{aligned} \tag{10}$$

Then, let $X_T = \ln F_T$, making use of a version of the Bayes' rule for (3.1) we obtain

$$\begin{aligned} E[e^{-\int_0^T r_t dt} S_T(F_T - K_F)^+|\mathcal{F}_T^U] &= e^{-\int_0^T r_t dt} E\left[S_T(e^{X_T} - e^{k_F})^+|\mathcal{F}_T^U\right] \\ &= e^{-\int_0^T r_t dt} E\left[S_T|\mathcal{F}_T^U\right] E^{Q_S}\left[(e^{X_T} - e^{k_F})^+|\mathcal{F}_T^U\right], \end{aligned} \tag{11}$$

where E^{Q_S} represents expectation under the measure Q_S.

The following proposition gives an integral expression for equity-linked foreign exchange option.

Proposition 1. *Under the Markovian regime-switching multi-scale jump-diffusion model, the price of equity-linked foreign exchange option is given by the following integral formula:*

$$C_F(0,T,K_F) = \frac{e^{-a_F k_F}}{\pi}\int_0^\infty e^{-iuk_F}\psi(0,T,u)du, \tag{12}$$

where

$$\begin{aligned} \psi(0,T,u) = &\frac{S_0\exp(g_0)\exp\left\{i(u - i(a_F+1))e^{-\beta T}X_0\right\}}{a_F^2 + a_F + i(2a_F+1)u - u^2} \\ &\times \left\langle U_0 \exp\left\{\int_0^T diag(\mathbf{g}(t,u))dt + \mathbf{A}T\right\}, \mathbf{1}\right\rangle, \end{aligned}$$

where $\mathbf{1} = \{1, 1, ..., 1\} \in \mathbf{R}^N$.

Write

$$\mathbf{g}(t,u) := (g_1(t,u), g_2(t,u), \cdots, g_N(t,u)) \in \mathfrak{C}^N,$$

where $\mathfrak{C}$ *is the complex space and* $\mathfrak{C}^N$ *is the* N*-fold product of* $\mathfrak{C}$*. For each* $j = 1, 2, \cdots, N$,

$$\begin{aligned} g_j(t,u) : &= i(u - i(a_F+1))e^{-\beta(T-t)}(\alpha_j + \gamma_j\rho_j\sigma_j - \frac{1}{2}\gamma_j^2) \\ &- r_j + \theta_j - \frac{1}{2}(u - i(a_F+1))^2 e^{-2\beta(T-t)\gamma_j^2}, \end{aligned} \tag{13}$$

$$\begin{aligned} g_0 &= \lambda_2 T\left(\int_0^T \phi_{Z^{(2)}}^{Q_S}\left((u - i(a_F+1))e^{-\beta(T-t)}\right)\frac{1}{T}dt - 1\right) \\ &+ \tilde{\lambda}_3 T\left(\int_0^T \phi_{Z^{(3)}}^{Q_S}\left((u - i(a_F+1))e^{-\beta(T-t)}\right)\frac{1}{T}dt - 1\right), \end{aligned} \tag{14}$$

$\tilde{\lambda}_3 = \lambda_3(k_3+1)$, *and the characteristic functions of* $Z^{(i)}$ *under the measure* Q_S *are given by* $\phi_{Z^{(i)}}^{Q_S}(u)$.

Before proving Proposition 1, we need to give some useful conclusions firstly.

Lemma 1. *The foreign exchange rate* F_t *satisfies the following stochastic differential equation given* $\mathcal{F}_T^U$ *under* Q_S

$$\frac{dF_t}{F_t} = (\alpha_t + \gamma_t\rho_t\sigma_t - \beta\ln F_t)dt + \gamma_t d\tilde{W}_t^{(2)} + (e^{Z_t^{(2)}} - 1)dN_t^{(2)} + (e^{Z_t^{(3)}} - 1)dN_t^{(3)}, \tag{15}$$

and

$$\tilde{W}_t^{(1)} = W_t^{(1)} - \int_0^t \sigma_s ds, \tilde{W}_t^{(2)} = W_t^{(2)} - \int_0^t \rho_s\sigma_s ds, \left\langle \tilde{W}_t^{(1)}, \tilde{W}_t^{(2)} \right\rangle = \int_0^t \rho_s ds. \tag{16}$$

The intensity of Poisson process $N_t^{(3)}$ *and the density function of* $Z^{(3)}$ *under the measure* Q_S *are given by*

$$\tilde{\lambda}_3 = \lambda_3(k_3+1), \tilde{f}_{Z^{(3)}}(z) = \frac{e^z f_{Z^{(3)}}(z)}{k_3+1}. \tag{17}$$

Proof. In light of (3.7), apply the Girsanov's theorem and by the independence of W and $N^{(i)}, i = 1, 2, 3$, we immediately get that $\tilde{W}_t^{(1)}$ and $\tilde{W}_t^{(2)}$ defined in (3.13) are also two standard Brownian motions with the instantaneous correlation coefficient at time t is still ρ_t. Also the Girsanov's theorem for point processes yields formula (3.14). As a consequence, the foreign exchange rate F_t under the measure Q_S obeys formula (3.12). Thus the proof of Lemma 1 is complete.

Because U, $N^{(i)}(i = 1, 2, 3)$ and $(W^{(1)}, W^{(2)})$ are mutually independent, the probability law of the Markov chain U remains the same after the measure change, i.e., under Q_S, U still has the semi-martingale dynamics.

Lemma 2. *N_t is a Poisson process with intensity λ. The jump amplitude of N_t is controlled by Z_t. For any time $t \neq s$, we assume that Z_t and Z_s are independently and identically distributed with $Z_t \sim f_Z(z)$. $T_k(k = 1, 2, \cdots, n)$ is the occurrence time of the kth event in n events and $\phi_{Z_k}(u)$ is the characteristic function of Z_k. We have the following conclusion*

$$E\left[\exp\left\{iu\int_0^T e^{-\beta(T-t)}Z_t dN_t\right\}\right] = \exp\left\{\lambda T\left(\int_0^T \phi_{Z_k}(ue^{-\beta(T-t)})\frac{1}{T}dt - 1\right)\right\}.$$

Proof. By the definition of the compound Poisson process we know

$$\begin{aligned}
E\left[\exp\left\{iu\int_0^T e^{-\beta(T-t)}Z_t dN_t\right\}\right] &= E\left[\exp\left\{\sum_{k=1}^{N_T} iue^{-\beta(T-T_k)}Z_k\right\}\right] \\
&= E\left[E\left[\exp\left\{\sum_{k=1}^{N_T} iue^{-\beta(T-T_k)}Z_k\right\}\right]\middle|\mathcal{F}_T^N\right] \\
&= E\left[\prod_{k=1}^{N_T} E\left[\exp\left\{iue^{-\beta(T-T_k)}Z_k\right\}\right]\middle|\mathcal{F}_T^N\right] \\
&= E\left[\prod_{k=1}^{N_T} \phi_{Z_k}(ue^{-\beta(T-T_k)})\right] \\
&= \sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}E\left[\prod_{k=1}^{n}\phi_{Z_k}(ue^{-\beta(T-T_k)})|N_T = n\right].
\end{aligned}$$

Moreover, Since N_T is a Poisson process, the times $T_k(k = 1, 2, \cdots, n)$ of the n events occurred are independent identically distributed and obey uniform distribution in $[0, T]$ regardless of the orders under the condition of $N_T = n$. Then

$$\begin{aligned}
&\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}E\left[\prod_{k=1}^{n}\phi_{Z_k}(ue^{-\beta(T-T_k)})|N_T = n\right] \\
&= \sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}\left[\prod_{k=1}^{n}E\left[\phi_{Z_k}(ue^{-\beta(T-T_k)})|N_T = n\right]\right] \\
&= \sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}\left(\prod_{k=1}^{n}\int_0^T \phi_{Z_k}(ue^{-\beta(T-t)})\frac{1}{T}dt\right) \\
&= \sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}\left(\int_0^T \phi_{Z_k}(ue^{-\beta(T-t)})\frac{1}{T}dt\right)^n \\
&= \exp\left\{\lambda T\left(\int_0^T \phi_{Z_k}(ue^{-\beta(T-t)})\frac{1}{T}dt - 1\right)\right\}.
\end{aligned}$$

The proof is completed.

Lemma 3. *Let* $\phi^{Q_S}_{X_T|\mathcal{F}^U_T}(u)$ *denote the conditional characteristic function of* X_T *given* $\mathcal{F}^U_T$ *under* Q_S*, which is defined as following*

$$\begin{aligned}\phi^{Q_S}_{X_T|\mathcal{F}^U_T}(u) &= E^{Q_S}[e^{iuX_T}|\mathcal{F}^U_T] \\ &= \exp\left\{iu\left(e^{-\beta T}X_0 + \int_0^T e^{-\beta(T-t)}(\alpha_t + \gamma_t\rho_t\sigma_t - \frac{1}{2}\gamma_t^2)dt\right)\right. \\ &- \frac{1}{2}u^2\int_0^T e^{-2\beta(T-t)}\gamma_t^2 dt + \lambda_2 T\left(\int_0^T \phi^{Q_S}_{Z^{(2)}}(ue^{-\beta(T-t)})\frac{1}{T}dt - 1\right) \\ &+ \left.\tilde{\lambda}_3 T\left(\int_0^T \phi^{Q_S}_{Z^{(3)}}(ue^{-\beta(T-t)})\frac{1}{T}dt - 1\right)\right\}.\end{aligned} \tag{18}$$

Proof. Note that $X_t = \ln F_t$ and (3.12), it is easy to obtain

$$dX_t = (\alpha_t + \gamma_t\rho_t\sigma_t - \frac{1}{2}\gamma_t^2 - \beta X_t)dt + \gamma_t d\tilde{W}_t^{(2)} + Z_t^{(2)}dN_t^{(2)} + Z_t^{(3)}dN_t^{(3)}.$$

Consequently,

$$\begin{aligned}X_T = e^{-\beta T}X_0 &+ \int_0^T e^{-\beta(T-t)}(\alpha_t + \gamma_t\rho_t\sigma_t - \frac{1}{2}\gamma_t^2)dt + \int_0^T e^{-\beta(T-t)}\gamma_t d\tilde{W}_t^{(2)} \\ &+ \int_0^T e^{-\beta(T-t)}Z_t^{(2)}dN_t^{(2)} + \int_0^T e^{-\beta(T-t)}Z_t^{(3)}dN_t^{(3)}.\end{aligned}$$

For convenient, we let

$$X_T = C_{1T} + C_{2T} + C_{3T},$$

where

$$C_{1T} = e^{-\beta T}X_0 + \int_0^T e^{-\beta(T-t)}(\alpha_t + \gamma_t\rho_t\sigma_t - \frac{1}{2}\gamma_t^2)dt + \int_0^T e^{-\beta(T-t)}\gamma_t d\tilde{W}_t^{(2)},$$

$$C_{2T} = \int_0^T e^{-\beta(T-t)}Z_t^{(2)}dN_t^{(2)}, \quad C_{3T} = \int_0^T e^{-\beta(T-t)}Z_t^{(3)}dN_t^{(3)}.$$

Then, due to $W^{(2)}, N^{(2)}, N^{(3)}, Z_t^{(2)}, Z_t^{(3)}$ are mutual independence, we can get

$$\begin{aligned}\phi^{Q_S}_{X_T|\mathcal{F}^U_T}(u) &= E^{Q_S}[e^{iu(C_{1T}+C_{2T}+C_{3T})}|\mathcal{F}^U_T] \\ &= E^{Q_S}[e^{iuC_{1T}}|\mathcal{F}^U_T] \times E^{Q_S}[e^{iuC_{2T}}|\mathcal{F}^U_T] \times E^{Q_S}[e^{iuC_{3T}}|\mathcal{F}^U_T].\end{aligned} \tag{19}$$

Furthermore, noting that the conditional distribution of C_{1T} is a normal distribution given $\mathcal{F}^U_T$. Hence, the characteristic function of C_{1T} can be obtained directly,

$$E^{Q_S}[e^{iuC_{1T}}|\mathcal{F}^U_T] = \exp\left\{iu\left(e^{-\beta T}X_0 + \int_0^T e^{-\beta(T-t)}(\alpha_t + \gamma_t\rho_t\sigma_t - \frac{1}{2}\gamma_t^2)dt\right)\right.$$

$$-\frac{1}{2}u^2\int_0^T e^{-2\beta(T-t)}\gamma_t^2 dt\Bigg\},$$

and, by Lemma 2

$$\begin{aligned}E^{Q_S}[e^{iuC_{2T}}|\mathcal{F}_T^U] &= E^{Q_S}[e^{iu\int_0^T e^{-\beta(T-t)}Z_t^{(2)}dN_t^{(2)}}|\mathcal{F}_T^U] \\ &= \exp\left\{\lambda_2 T\left(\int_0^T \phi_{Z^{(2)}}^{Q_S}(ue^{-\beta(T-t)})\frac{1}{T}dt - 1\right)\right\},\end{aligned}$$

$$\begin{aligned}E^{Q_S}[e^{iuC_{3T}}|\mathcal{F}_T^U] &= E^{Q_S}[e^{iu\int_0^T e^{-\beta(T-t)}Z_t^{(3)}dN_t^{(3)}}|\mathcal{F}_T^U] \\ &= \exp\left\{\tilde{\lambda}_3 T\left(\int_0^T \phi_{Z^{(3)}}^{Q_S}(ue^{-\beta(T-t)})\frac{1}{T}dt - 1\right)\right\}.\end{aligned}$$

Finally, it follows from formula (3.16) and the above three parts of the characteristic function, we can get the result. The proof is therefore complete.

Lemma 4. *The Fourier transform of the equity-linked foreign exchange call option price is given by*

$$\begin{aligned}\psi(0,T,u) = &\frac{S_0\exp(g_0)\exp\left\{i(u-i(a_F+1))e^{-\beta T}X_0\right\}}{a_F^2+a_F+i(2a_F+1)u-u^2} \\ &\times\left\langle U_0\exp\left\{\int_0^T diag(\mathbf{g}(t,u))dt+\mathbf{A}T\right\},\mathbf{1}\right\rangle.\end{aligned}$$

Proof. Let $f_{X_T|\mathcal{F}_T^U}^{Q_S}(x)$ and $\phi_{X_T|\mathcal{F}_T^U}^{Q_S}(u)$ denote the conditional distribution function and the conditional characteristic function of X_T given $\mathcal{F}_T^U$ under Q_S, respectively. Then by direct calculation we have

$$\begin{aligned}\psi(0,T,u) =& \int_{-\infty}^{+\infty} e^{iuk_F}c(0,T,k_F)dk_F \\ =& \int_{-\infty}^{+\infty} e^{iuk_F}e^{a_Fk_F}E\left[e^{-\int_0^T r_t dt}S_T(F_T-K_F)^+\right]dk_F \\ =&E\left[\int_{-\infty}^{+\infty} e^{iuk_F}e^{a_Fk_F}E\left[e^{-\int_0^T r_t dt}S_T(e^{X_T}-e^{k_F})^+|\mathcal{F}_T^U\right]dk_F\right] \\ =&S_0E\left[e^{\int_0^T(-r_t+\theta_t)dt}\int_{-\infty}^{+\infty} e^{iuk_F}e^{a_Fk_F}E^{Q_S}\left[(e^{X_T}-e^{k_F})^+|\mathcal{F}_T^U\right]dk_F\right] \\ =&S_0E\left[e^{\int_0^T(-r_t+\theta_t)dt}\int_{-\infty}^{+\infty} e^{iuk_F}e^{a_Fk_F}\int_{k_F}^{+\infty}(e^{X_T}-e^{k_F})f_{X_T|\mathcal{F}_T^U}^{Q_S}(x)dxdk_F\right] \\ =&S_0E\left[e^{\int_0^T(-r_t+\theta_t)dt}\int_{-\infty}^{+\infty}\int_{-\infty}^{x}(e^{x+(a_F+iu)k_F}\right. \\ &\left.-e^{(1+a_F+iu)k_F})dk_Ff_{X_T|\mathcal{F}_T^U}^{Q_S}(x)dx\right]\end{aligned}$$

$$
\begin{aligned}
&= S_0 E\left[e^{\int_0^T(-r_t+\theta_t)dt}\int_{-\infty}^{+\infty}\left(\frac{e^{(1+a_F+iu)x}}{a_F^2+a_F+i(2a_F+1)u-u^2}\right)f^{Q_S}_{X_T|\mathcal{F}^U_T}(x)dx\right]\\
&= \frac{S_0 E\left[e^{\int_0^T(-r_t+\theta_t)dt}\phi^{Q_S}_{X_T|\mathcal{F}^U_T}(u-i(a_F+1))\right]}{a_F^2+a_F+i(2a_F+1)u-u^2}. \qquad (20)
\end{aligned}
$$

Furthermore, according to Lemma 3, we can obtain

$$
\begin{aligned}
&\phi^{Q_S}_{X_T|\mathcal{F}^U_T}(u-i(a_F+1))\\
= \ & \exp\Big\{i(u-i(a_F+1))\left(e^{-\beta T}X_0+\int_0^T e^{-\beta(T-t)}(\alpha_t+\gamma_t\rho_t\sigma_t-\frac{1}{2}\gamma_t^2)dt\right)\\
- \ & \frac{1}{2}(u-i(a_F+1))^2\int_0^T e^{-2\beta(T-t)}\gamma_t^2\Big\}\\
\times \ & \exp\Big\{\lambda_2 T\left(\int_0^T \phi^{Q_S}_{Z^{(2)}}((u-i(a_F+1))e^{-\beta(T-t)})\frac{1}{T}dt-1\right)\\
+ \ & \tilde{\lambda}_3 T\left(\int_0^T \phi^{Q_S}_{Z^{(3)}}((u-i(a_F+1))e^{-\beta(T-t)})\frac{1}{T}dt-1\right)\Big\}.
\end{aligned}
$$

On the other hand, using (3.10), (3.11) and

$$
\mathbf{g}(t,u) := (g_1(t,u), g_2(t,u),\cdots,g_N(t,u))' \in \mathfrak{C}^N,
$$

we have

$$
\begin{aligned}
&E\left[e^{\int_0^T(-r_t+\theta_t)dt}\phi^{Q_S}_{X_T|\mathcal{F}^U_T}(u-i(a_F+1))\right]\\
&= \exp\left\{i(u-i(a_F+1))e^{-\beta T}X_0+g_0\right\}E\left[\exp\left(\int_0^T <\mathbf{g}(t,u),U_t> dt\right)\right]. \qquad (21)
\end{aligned}
$$

Refer to Lemma 3.3 in [15], we know

$$
E\left[\exp\left(\int_0^T <\mathbf{g}(t,u),U_t> dt\right)\right] = \left\langle U_0\exp\left\{\int_0^T diag(\mathbf{g}(t,u))dt+\mathbf{A}T\right\},\mathbf{1}\right\rangle. \quad (22)
$$

Finally, by (3.18) and (3.19), the conclusion is established.

From the above conclusions, we can obtain the results of Proposition 1 directly.

Proof of Proposition 1. The proof is standard. Applying the inverse Fourier transform to formula (3.3), the following equation can be derived:

$$
\begin{aligned}
C(0,T,K_F) &= e^{-a_F k_F}c(0,T,k_F)\\
&= e^{-a_F k_F}\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-iuk_F}\psi(0,T,u)du\\
&= e^{-a_F k_F}\frac{1}{\pi}\int_0^{+\infty}e^{-iuk_F}\psi(0,T,u)du.
\end{aligned}
$$

Hence we obtain the result from Lemma 4.

4. NUMERICAL EXAMPLES

In this section, we perform a numerical analysis for equity-linked foreign exchange call option pricing formula (3.9) by using fast Fourier transform algorithm (see [19], [21] and [22]). For convenience, all jump amplitudes in this paper are assumed to be log-double-exponential distributions, and the pricing model given in this paper can also be applied to other jump size distribution.

We assume that $Z_t^{(i)} (i = 1, 2, 3)$ obeys a double exponential distribution with the following density function

$$f_{Z^{(i)}}(z) = p_i \eta_{i1} e^{-\eta_{i1} z} 1_{\{z \geq 0\}} + q_i \eta_{i2} e^{\eta_{i2} z} 1_{\{z<0\}},$$

$$\eta_{i1} > 1, \eta_{i2} > 0, p_i \geq 0, q_i \geq 0, p_i + q_i = 1.$$

By the characteristic function of a double exponential distribution, we know that

$$\phi_{Z^{(i)}}(ue^{-\beta(T-t)}) = \frac{p_i \eta_{i1}}{\eta_{i1} - iue^{-\beta(T-t)}} + \frac{q_i \eta_{i2}}{\eta_{i2} + iue^{-\beta(T-t)}}.$$

Through the integral calculation we can get the following result directly

$$\begin{aligned}
&\int_0^T \phi_{Z^{(i)}}(ue^{-\beta(T-t)}) \frac{1}{T} dt \\
&= \frac{1}{T} \int_0^T \left(\frac{p_i \eta_{i1}}{\eta_{i1} - iue^{-\beta(T-t)}} + \frac{q_i \eta_{i2}}{\eta_{i2} + iue^{-\beta(T-t)}} \right) dt \\
&= p_i - \frac{p_i}{\beta T} \ln \left(\frac{\eta_{i1} - iu}{\eta_{i1} - iue^{-\beta T}} \right) + q_i - \frac{q_i}{\beta T} \ln \left(\frac{\eta_{i2} + iu}{\eta_{i2} + iue^{-\beta T}} \right).
\end{aligned}$$

Consequently,

$$\begin{aligned}
&\exp\left\{ \lambda_i T \left(\int_0^T \phi_{Z^{(i)}}(ue^{-\beta(T-t)}) \frac{1}{T} dt - 1 \right) \right\} \\
&= \exp\left\{ \lambda_i T \left(-\frac{p_i}{\beta T} \ln \left(\frac{\eta_{i1} - iu}{\eta_{i1} - iue^{-\beta T}} \right) - \frac{q_i}{\beta T} \ln \left(\frac{\eta_{i2} + iu}{\eta_{i2} + iue^{-\beta T}} \right) \right) \right\} \\
&= \exp\left\{ \ln \left(\frac{\eta_{i1} - iu}{\eta_{i1} - iue^{-\beta T}} \right)^{-\frac{\lambda_i p_i}{\beta}} \right\} \exp\left\{ \ln \left(\frac{\eta_{i2} + iu}{\eta_{i2} + iue^{-\beta T}} \right)^{-\frac{\lambda_i q_i}{\beta}} \right\} \\
&= \left(\frac{\eta_{i1} - iu}{\eta_{i1} - iue^{-\beta T}} \right)^{-\frac{\lambda_i p_i}{\beta}} \times \left(\frac{\eta_{i2} + iu}{\eta_{i2} + iue^{-\beta T}} \right)^{-\frac{\lambda_i q_i}{\beta}}.
\end{aligned}$$

By Lemma 1, we can get

$$\begin{aligned}
\tilde{f}_{Z^{(i)}} &= \frac{e^z f(z)}{k_i + 1} \\
&= \frac{p_i \eta_{i1} e^{-(\eta_{i1}-1)z} 1_{\{z \geq 0\}}}{k_i + 1} + \frac{q_i \eta_{i2} e^{(\eta_{i2}+1)z} 1_{\{z<0\}}}{k_i + 1}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{p_i \eta_{i1}}{(k_i+1)(\eta_{i1}-1)}(\eta_{i1}-1)e^{-(\eta_{i1}-1)z}1_{\{z\geq 0\}} \\
&\quad + \frac{q_i \eta_{i2}}{(k_i+1)(\eta_{i2}+1)}(\eta_{i2}+1)e^{(\eta_{i2}+1)z}1_{\{z<0\}} \\
&= \tilde{p}_i \tilde{\eta}_{i1} e^{-\tilde{\eta}_{i1} z}1_{\{z\geq 0\}} + \tilde{q}_i \tilde{\eta}_{i2} e^{\tilde{\eta}_{i2} z}1_{\{z<0\}},
\end{aligned}
$$

where

$$
\tilde{p}_i = \frac{p_i \eta_{i1}}{(k_i+1)(\eta_{i1}-1)}, \tilde{q}_i = \frac{q_i \eta_{i2}}{(k_i+1)(\eta_{i2}+1)}, \tilde{\eta}_{i1} = \eta_{i1}-1, \tilde{\eta}_{i2} = \eta_{i2}+1.
$$

So

$$
\begin{aligned}
&\exp(g_0) \\
&= \left(\frac{\eta_{21} - i(u-i(a_F+1))}{\eta_{21} - i(u-i(a_F+1))e^{-\beta T}}\right)^{-\frac{\lambda_2 p_2}{\beta}} \times \left(\frac{\eta_{22} + i(u-i(a_F+1))}{\eta_{22} + i(u-i(a_F+1))e^{-\beta T}}\right)^{-\frac{\lambda_2 q_2}{\beta}} \\
&\times \left(\frac{\tilde{\eta}_{31} - i(u-i(a_F+1))}{\tilde{\eta}_{31} - i(u-i(a_F+1))e^{-\beta T}}\right)^{-\frac{\tilde{\lambda}_3 \tilde{p}_3}{\beta}} \times \left(\frac{\tilde{\eta}_{32} + i(u-i(a_F+1))}{\tilde{\eta}_{32} + i(u-i(a_F+1))e^{-\beta T}}\right)^{-\frac{\tilde{\lambda}_3 \tilde{q}_3}{\beta}},
\end{aligned}
$$

where

$$
\tilde{p}_3 = \frac{p_3 \eta_{31}}{(k_3+1)(\eta_{31}-1)}, \tilde{q}_3 = \frac{q_3 \eta_{32}}{(k_3+1)(\eta_{32}+1)}, \tilde{\eta}_{31} = \eta_{31}-1, \tilde{\eta}_{32} = \eta_{32}+1,
$$

$$
\tilde{\lambda}_3 = \lambda_3(k_3+1), k_3 = E(e^{Z_t^{(3)}}-1) = p_3\frac{\eta_{31}}{\eta_{31}-1} + q_3\frac{\eta_{32}}{\eta_{32}+1} - 1.
$$

To simplify our computation, we parsimoniously assume that Markov chain U has two states, i.e., $N = 2$. The first and second regimes, namely $e_1 = (1, 0)$ and $e_2 = (0, 1)$, can be interpreted as a good economic state (state 1) and a bad economic state (state 2), respectively. The preference parameters listed in Table 1 depend on the Markov chain. For instance, r_1 and r_2 denote the interest rates in a good state and a bad state, respectively. Besides, for space consideration, we do not give the value of parameters associate with the jumps and these parameters are shown here. For $Z_t^{(2)}$, we let $\eta_{21} = 5.1$, $\eta_{22} = 2.5$, $p_2 = 0.5$, $q_2 = 0.5$. For $Z_t^{(3)}$, we let $\eta_{31} = 4$, $\eta_{32} = 2$, $p_3 = 0.25$, $q_3 = 0.75$. We assume $S_0 = 1$, $F_0 = 1$, $\beta = 1$, $\lambda_2 = 1$, $\lambda_3 = 1$, $T = 1$. We also assume $a_{ii} = -a$ in the rate matrix of U under Q, that is

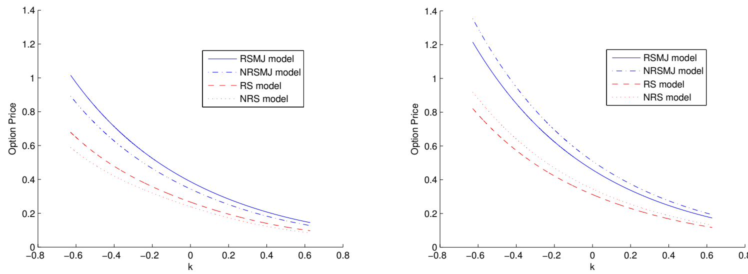
$$
A = \begin{pmatrix} -a & a \\ a & -a \end{pmatrix},
$$

where a takes values in $[0, 1]$.

Here the FFT method is applied to calculate the equity-linked foreign exchange call option price under the regime-switching multi-scale jump-diffusion model. The impact of different models and different parameters on option prices are analyzed in Figures 1-4.

Parameter name	*Value in state 1*	*Value in state 2*
Domestic interest rate	$r_1 = 0.04$	$r_2 = 0.02$
Risk-neutral parameter of S	$\theta_1 = 0.04$	$\theta_2 = 0.02$
Mean-reversion level of F	$\alpha_1 = 0.04$	$\alpha_2 = 0.02$
Volatility of F	$\gamma_1 = 0.6$	$\gamma_2 = 0.8$
Correlation coefficient	$\rho_1 = 0.2$	$\rho_2 = 0.4$
Volatility of S	$\sigma_1 = 0.2$	$\sigma_2 = 0.4$

Table 1: Parameters that depend on the Markov chain



(a) state 1 (b) state 2

Figure 1: Option prices calculated under the RSMJ, NRSMJ, RS, NRS models in state 1 and state 2.

Figure 1 shows the option prices calculated under the proposed models in state 1 corresponding to subgraph (a) and state 2 corresponding to subgraph (b) with the assumption that $a = 0.5$. In Figure 1, the solid lines, dot-dashed lines, dashed lines, and dotted lines correspond to the regime-switching multi-scale jump-diffusion model (RSMJ), the model without regime-switching (NRSMJ), the model without jump-diffusion (RS) and to the model without regime-switching and jump-diffusion (NRS). Our choice of the four reference models is in order to observe the impact of regime-switching and jump-risk on equity-linked foreign exchange option prices. From Figure 1(a) and Figure 1(b), we can see that the option prices under different models decreases with the increase of logarithmic currency strike k. The option prices in state 1 are systematically lower than those in state 2 when k is fixed. If the option valuation is viewed from the perspective of a domestic investor, state 1 is a "Good" state with a higher interest and a lower volatility than those in state 2 which is considered to be a "Bad" one. A "Good" state means less chance of the equity price being very high or very low. In this case, the option will be less valuable. Consequently, it is reasonable that the option prices in state 1 are lower than the corresponding prices in State 2. This is in conformity with the view of [15].

Figure 1 provides us with avisual multiple comparison among the option prices under the RSMJ model, NRSMJ model, RS model and NRS model, with $a = 0.5$ in the RSMJ model and RS model. As indicated in Figure 1, the equity-linked foreign exchange option prices of RSMJ model and RS model are higher (lower) than those of NRSMJ model and NRS model in state 1 (state 2). In other words, ignoring the regime-switching effect would result in the option prices being underpriced in state 1 and being overpriced in state 2. Generally the option prices with jumps are higher than those of the models without jumps.

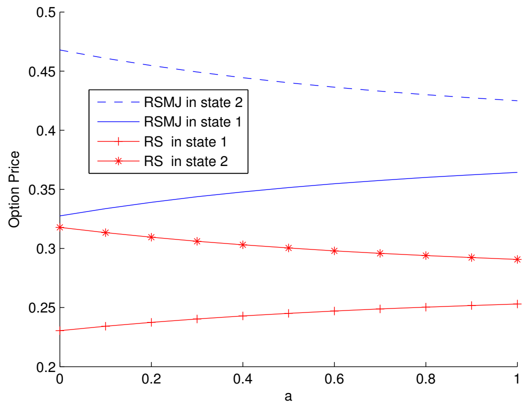
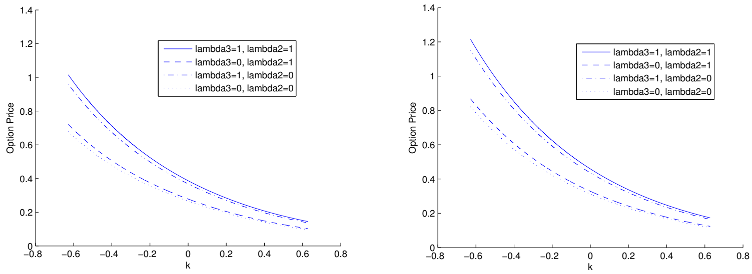


Figure 2: Option prices corresponding to different a with $k = 0$ under RSMJ model and RS model.

Figure 2 shows the effect of a on option prices under the RSMJ model and RS model with fixed $k = 0$. The solid line, dashed line, "+" line and "*" line correspond to RSMJ model in state 1, RSMJ model in state 2, RS model in state 1 and RS model in state 2. From Figure 2, we can see that when a increases, the option prices under RSMJ model (RS model) in state 1 and state 2 display different trends. In state 1, the option prices increase while decrease with a in state 2. When $a = 0$, the regime-switching effect disappears and the option prices are the highest in state 2 and lowest in state 1. As in the previous analysis, the option prices in state 1 are more cheaper than those in state 2 and the option prices under RSMJ model are more expensive than those under RS model.

In Figure 3, in order to consider the option prices against the common jump intensity λ_3 and individual jump intensity of F, λ_2, four special cases of RSMJ model with $a = 0.5$ are taken into account. The dotted lines, dashed lines, dot-dashed lines and solid lines correspond to the four cases of $\lambda_2 = 0, \lambda_3 = 0$; $\lambda_2 = 1, \lambda_3 = 0$; $\lambda_2 = 0, \lambda_3 = 1$ and $\lambda_2 = 1, \lambda_3 = 1$. In both state 1 corresponding to Figure 3(a) and state 2 corresponding to Figure 3(b), We can see that the option prices increase



(a) state 1 (b) state 2

Figure 3: Option prices calculated with different values of λ_2 and λ_3 under the RSMJ models in state 1 and state 2.

when considering the four cases in order. Figure 3 also shows that the trend of the two states is consistent although the numerical values are different. In addition, we also find that the option prices are higher when considering the individual jump and common jump together than those of considering the individual jump or considering the common jump separately. In particular, the influence of λ_3 is more significant than λ_2. The reason is that common jump is not only related to the foreign equity but also to the exchange rate. However, λ_2 is only related to the exchange rate itself and it's impact is more weaker. Hence, λ_3 plays a more prominent role in option pricing. In order to illustrate the influence between λ_2 and λ_3 more clearly, Figure 4 provides the effects of λ_2 and λ_3 on option prices in state 1 and state 2 with $k = 0$, $a = 0.5$ under RSMJ model. The solid line (the "+" line) denotes the option prices in state 1 (state 2) according to λ_3 with $k = 0, \lambda_2 = 1$. The dashed line (the "*" line) denotes the option prices in state 1 (state 2) according to λ_2 with $k = 0, \lambda_3 = 1$. From Figure 4, we can see that the option prices increase when λ_2 (λ_3) increase with fixed $\lambda_3 = 1$ ($\lambda_2 = 1$), but the increase speed of λ_3 is higher than that of λ_2 in state 1 and state 2. It can be seen that it is meaningful to consider the multi-scale jump-diffusion model.

5. CONCLUSION

This paper proposes equity-linked foreign exchange call option pricing problem under the regime-switching multi-scale jump-diffusion model. The financial model used in this paper which can reflect the characteristics of regime-switching, mean reversion, jump-diffusion for foreign exchange rate. By the measure change and Fourier transform technique, we obtain the explicit expression of price of the equity-linked foreign exchange call option. One novelty of this paper is that we introduce the regime-switching mean reversion multi-scale jump-diffusion process to describe the

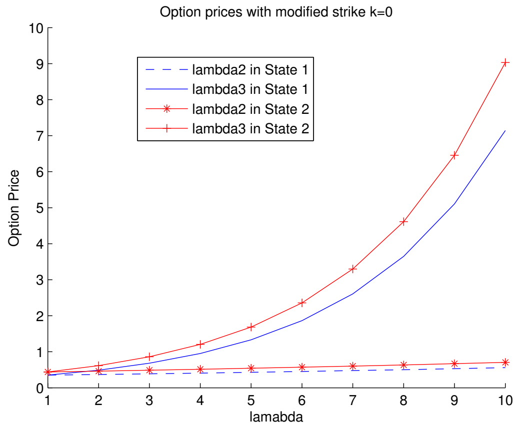


Figure 4: Option prices corresponding to different λ_2 and λ_3 with $k = 0$.

foreign exchange rate. We extend the work of [15]. Numerical results reveal that regime switching and jump components have significant impaction on the price of the equity-linked foreign exchange call option.

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