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EXISTENCE AND ASYMPTOTIC STABILITY OF AN IMPULSIVE STOCHASTIC DIFFERENTIAL EQUATION

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ABSTRACT. In this paper, we give necessary conditions for the existence and asymptotic stability of a mild solution for the impulsive stochastic differential equation. It is shown that the impulsive stochastic differential equation has a mild solution and the solution is asymptotically stable in the p-th moment.

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1. INTRODUCTION

Stochastic differential equations have been widely applied in science, engineering, biology, mathematical finance and in almost all sciences. In the current literature, there are many papers on the existence and uniqueness of solutions to stochastic differential equations see [9,5,6] and references therein. More recently, Fu and Liu [4] discussed the existence and uniqueness of square-mean almost automorphic solutions to some linear and nonlinear stochastic differential equations, the asymptotic stability of the unique square-mean almost automorphic solution was established in the square-mean sense.

Impulsive differential equations model problem with impulsive effects which are due to instantaneous perturbations at certain moments. The vast applications of the theory of impulsive differential equations and inclusions have attracted many authors to considering both deterministic and stochastic cases. The theory of impulsive differential equations were extensively studied in [6] and [2] for instance, while Pan [10] considered the existence of mild solution for impulsive stochastic differential equations with nonlocal conditions in $\mathcal{PC}$-norm.

Correspondingly, a lot of stability results of impulsive differential equations have been obtain [1,12,13,7]. In particular, Liu [8] established comparison principles of existence and uniqueness and stability of solutions for impulsive differential systems by means of Lyapunov function method and Ito's formula. Peng and Jia [11] obtained some criteria on p-th moment stability and p-th moment asymptotical stability of impulsive stochastic functional differential equations by using Lyapunov-Razumikhin

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method. In [3], several criteria on the global exponential stability and instability of impulsive stochastic functional differential systems are obtained by Cheng and Deng. Inspired by [10] and [11], we extend the result to mild solutions of impulsive stochastic differential equations with local conditions and study its asymptotic stability in the p-th moment. In the sequel, preliminaries necessary for the result shall be stated in section 2 and the main result will be proved in section 3.

2. PRELIMINARIES

Let (Ω, Γ, P) be a complete probability space with probability measure P on Ω and a normal filtration $\{\Gamma_t\}_{t\geq 0}$. Let X, Y be two real separable Hilbert spaces with norms $\|\cdot\|_X, \|\cdot\|_Y$ and Q-Wiener process on (Ω, Γ, P) with covariance operator $Q \in \mathcal{BL}(Y)$ such that $trQ < \infty$. Let $\mathcal{L}(X, Y)$ be the space of bounded linear operators mapping X into Y equipped with the usual norm $\|\cdot\|$. We assume that there exist a complete orthonormal system $e_{ii\geq 1}$ in Y, a bounded sequence of nonnegative real numbers λ_i such that $Qe_i = \lambda_i e_i$, $i = 1, 2, \ldots$, and a sequence β_i $i \geq 1$ of independent Brownian motions such that $\langle w(t), e\rangle = \sum \sqrt{\lambda_i}\,\langle e_i, e\rangle\, \beta_i(t)$, $e \in Y$, and $\Gamma_t = \Gamma_t^w$, where Γ_t^w is the sigma algebra generated by $w(s) : 0 \leq s \leq t$. Let $\mathcal{L}_2^0 = \mathcal{L}_2(Q^{1/2}Y; X)$ be the space of all Hilbert-Schmidt operators from $Q^{1/2}Y$ to X with the inner product $\langle \mu, \xi\rangle_{L_2^0} = tr[\mu Q \xi]$.

We consider the existence of mild solution for the following impulsive stochastic differential equations in a Hilbert space

(2.1)
$$\begin{cases} dx(t) = [Ax(t) + F(t, x(t))]dt + G(t, x(t))dW(t), & t \geq 0, \quad t \neq t_k \\ \Delta x(t_k) = I_k(x(t_k)), & t = t_k, \quad k = 1, 2, \ldots, m, \,. \\ x(0) = x_0 \end{cases}$$

where $A : D(A) \subseteq X \to X$ is the infinitesimal generator of strongly continuous semigroup of bounded linear operators $T(t)$, $t > 0$.

X is a real Banach space. $x(0) = x_0 \in X$, $G : [0, b] \to X$. $F : [0, b] \times X \to X$; let $0 < t_1 < \cdots < t_m < t_{m+1} = b$. $I_k : X \to X$, where $k = 1, \ldots, m$ are impulsive functions, $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$ which is the right and left limit of x at t_k. F and G are predictable processes with Bochner integrable trajectories on arbitrary finite interval $[0, b]$.

Definition 2.1. The stochastic process $x(t)$, $t \in [0, b] \to X$ is called the mild solution for the impulsive SDE (2.1) if

(i) $x(t)$ is adapted to Γ_t, $t \geq 0$.
(ii) $x(t) \in X$ has cadlag paths on $t \in [0, b]$ a.s and for each $t \in [0, b]$.
(iii) For an arbitrary t $\in [0, b]$.

$$(2.2)\quad \begin{cases} x(t) = & T(t)x_0 + \int_0^t T(t-s)F(s,x(s))ds + \int_0^t T(t-s)G(s,x(s))dW(s) \\ & + \sum_{0<t_k<t} T(t-t_k)I_k(x(t_k)). \end{cases}$$

3. MAIN RESULTS

The following important theorem and assumptions are used to obtain the existence of (2.1)

Theorem 3.1. *Let $F : [0,b] \times X \to X$ be an L^1-Caratheodory function and $G : [0,b] \times X \to X$ satisfying the following conditions:*

(a) *For each $t \in [a,b]$, $G(t,\cdot) : X \to X$ is continuous for all $x_0 \in X$, $G(\cdot,x) : [0,b] \to X$ is measurable.*
(b) *The function $G : [0,b] \times X \to X$ satisfies (i) and there exist $L_G > 0$ such that for $0 \leq s_1, s_2 \leq T$, $x_i, y_i \in X$, $i = 0,1,2\ldots.$ $\|G(s_1,x_0) - G(s_2,y_0)\|^p_{L_2^0} \leq L_G(\|s_1 - s_2\|^p + \max\|x_i - y_i\|^p)$.*
(c) *For any $l > 0$ there exist a function $\rho_l \in L^1(0,b)$ such that $\sup E\,\|G(t,x)\|^p_{L_2^0} \leq \rho_l(t)\,\|x\|^p \leq l$ and $\liminf_{l\to+\infty} \frac{1}{l}[\int_0^T \rho_l(s)^{\frac{2}{p}} ds]^{\frac{p}{2}} = \eta < \infty$.*

Also assume that

(i) *there exist constant C_k such that $\|I_k(x)\| \leq C_k$, $k = 1,2,\ldots,m$ for each $x \in X$,*
(ii) *there exist a constant M such that $\|T(t)\|_{B(E)} \leq M$ for each $t \geq 0$,*
(iii) *there exist a continuous nondecreasing function $\Psi : [0,\infty) \to [0,\infty)$ and $p \in L^1(0,b;\Re_+)$ such that $|F(t,x)| \leq p(t)\Psi(|x|)$, for a.e $t \in [0,b]$ and each $x \in X$, with*

$$\int_0^b m(s)ds < \int_C^\infty \frac{dx}{x + \Psi(x)},$$

where

$$m(s) = \max\left\{M\|B\|_{B(E)}, Mp(s)\right\}, \quad c = M\left[\|x_0\| + \sum_{k=1}^m c_k\right],$$

(iv) *For each bounded $\mathcal{B} \subseteq \mathcal{PC}(0,b;X)$ and $t \in [0,b]$, the set*

$$\Bigg\{T(t)x_0 + \int_0^t T(t-s)F(s,x(s))ds + \int_0^t T(t-s)G(s,x(s))dW$$
$$+ \sum_{0<t_k<t} T(t-t_k)I_k(x(t_k^-)) : x \in \mathcal{B}\Bigg\}$$

is relatively compact in X, then the impulsive SDE (2.1) has at least one mild solution.

Proof. Transforming the problem (2.2) into a fixed point problem. Consider the operator $\Phi : \mathcal{PC}(0, b; X) \to \mathcal{PC}(0, b; X)$ defined by

$$\begin{cases} \Phi(x)(t) & = T(t)x_0 + \int_0^t T(t-s)F(s, x(s))ds \\ & \quad + \int_0^t T(t-s)G(s, x(s))dW(s) + \sum_{0<t_k<t} T(t-t_k)I_k(x(t_k^-)) \, . \end{cases} \tag{3.1}$$

Clearly, the fixed point of Φ are mild solutions to the SDE (2.1) Subsequently, we will prove that Φ has a fixed point by Schaefer's fixed point theorem.

The proof will be given in several steps.

Step 1: Φ is continuous

Let $\{x_n\}_{n=1}^{\infty}$ be a sequence in $\mathcal{PC}(0, b; X)$ such that $x_n \to x$. We will show that $\Phi(x_n) \to \Phi(x)$. For each $t \in [0, b]$, we have

$$\begin{aligned} \Phi(x_n)(t) = T(t)x_0 + \int_0^t T(t-s)F(s, x_n(s))ds + \int_0^t T(t-s)G(s, x_n(s))dW(s) \\ + \sum_{0<t_k<t} T(t-t_k)I_k(x_n(t_k)) \, . \end{aligned}$$

Then,

$$\begin{aligned} E\|\Phi(x_n)(t) - \Phi(x)(t)\| &\leq \sup_{t\in[0,b]} E \left\| \int_0^t T(t-s)[F(s, x_n(s)) - F(s, x(s))]ds \right\| \\ &\quad + \sup_{t\in[0,b]} E \left\| \int_0^t T(t-s)[G(s, x_n(s)) - G(s, x(s))]dW \right\| \\ &\quad + \sup_{t\in[0,b]} E \left\| \sum_{0<t_k<t} T(t-t_k)I_k x_n(t_k) - I_k x(t_k) \right\| \\ &\leq M \int_0^t \|F(s, x_n(s)) - F(s, x(s))\| \, ds \\ &\quad + M \int_0^t \|G(s, x_n(s)) - G(s, x(s))\| \, dW \\ &\quad + M \sum_{0<t_k<t} \sup_{t\in[0,b]} E \, \|I_k x_n(t_k) - I_k x(t_k)\| \end{aligned}$$

Since I_k, where $k = 1, 2, \ldots m$ are continuous, and $\lim_{n\to\infty} E \, \|\Phi x_n - \Phi x\| \to 0$ this implies that Φ is continuous.

Step 2: Φ maps bounded sets into bounded sets in $\mathcal{PC}(0, b; X)$.

It is enough to show that for any $q > 0$, there exists a $\delta > 0$ such that for each $x \in \mathcal{B}_q = \{y \in \mathcal{PC}(0, b; X) : \|x\|_{\mathcal{PC}} \leq q\}$, one has $\|\Phi(x)\|_{\mathcal{PC}} \leq \delta$.

By assumptions (i)–(ii) and the fact that F is L^1-Caratheodory function, we have, for each $t \in [0,b]$,

$$E\|\Phi(x)(t)\| \leq M\|x_0\| + M\int_0^t \|\varphi_q(s)\|\,ds + M\sum_{k=1}^m C_k$$
$$\leq M\|x_0\| + M\|\varphi_q\|_{L^1} + M\sum_{k=1}^m C_k := \delta$$

Step 3: Φ maps bounded sets into equicontinuous sets of $\mathcal{PC}(0,b;X)$.

Let $x \in \mathcal{PC}(0,b;X), t_1 \geq 0$ and ϵ be sufficiently small, then

$$\begin{aligned} E\|\Phi(x)(t_1+\epsilon) - \Phi(x)(t_1)\| \leq \|T(t_1+\epsilon) - T(t_1)\|\,\|x_0\| \\ + \int_0^{t_1} E\|T(t_1+\epsilon-s) - T(t_1-s)F(s,x(s))\|\,ds \\ + \int_{t_1}^{t_1+\epsilon} E\|T(t_1+\epsilon-s)F(s,x(s))\|\,ds \\ + \int_0^{t_1} E\|T(t_1+\epsilon-s) - T(t_1-s)G(s,x(s))\|\,dW(s) \\ + \int_{t_1}^{t_1+\epsilon} E\|T(t_1+\epsilon-s)G(s,x(s))\|\,dW(s) \\ + \sum_{t_1<t<t_1+\epsilon} C_k|T(t_1+\epsilon-t_k) - T(t_1-t_k)I_k x(t_k)| \end{aligned}$$

and $E\|\Phi(x)(t_1+\epsilon) - \Phi(x)(t_1)\| \to 0$ as $\epsilon \to 0$.

As a consequence of Steps 1 to 3 and asumption (iv) of Theorem 3.1 together with the Arzela-Ascoli theorem, we can deduce that $\Phi : \mathcal{PC}(0,b;X) \to \mathcal{PC}(0,b;X)$ is a completely continuous operator.

Step 4: Now we show that the set $\xi(\Phi) := \{x \in \mathcal{PC}(0,b;X) : x = \lambda\Phi(x),\ 0 < \lambda < 1\}$ is bounded.

Let $x \in \xi(\Phi)$, then $x = \lambda\Phi(x)$, for some $0 < \lambda < 1$. Thus, for each $t \in [0,b]$,

$$x(t) = \lambda\Bigg[T(t)x_0 + \int_0^t T(t-s)f(s,x(s))ds + \int_0^t T(t-s)g(s,x(s))dW$$
$$+ \sum_{0<t_k<t} T(t-t_k)I_k(y(t_k^-))\Bigg].$$

This implies by assumption (i) to (iii) that for each $t \in [0,b]$,

$$\|x(t)\| \leq M\|x_0\| + \int_0^t [m(\|x(s)\|)]ds + \int_0^t [\psi(\|x(s)\|)]dW + M\sum_{k=1}^m C_k.$$

Let us denote the right hand side of the above inequality by $v(t)$, then we have

$$|x(t)| \leq v(t) \text{ for every } t \in [0,b]$$

$$v(0) = M[\|x_0\|] + \sum_{k=1}^{m} C_k$$

$$v'(t) = \psi(\|x(t)\|) + m(\|x(t)\|), \text{ for a.e } t \in [0, b].$$

Using the increasing character of ψ, we get $v'(t) = \psi(v(t)) + M(v(t))$ for a.e $t \in [0, b]$. This shows that $\epsilon(\Phi)$is bounded. □

As a consequence of Schaefer's fixed point theorem [14], we deduce that Φ has a fixed point which is a mild solution of eqn (2.1), hence proved.

Asymptotic Stability of the Impulsive Stochastic Differential Equations.

Lemma 3.2. *For any $r \geq 1$ and for arbitrary L_2^0-valued predictable process $\Phi(\cdot)$* $\sup_{s\in[0,t]} E\left\|\int_0^s \Phi(u)dW(u)\right\|_X^{2r} \leq (r(2r-1))^r \left(\int_0^t (E\|\Phi(s)\|_{L_2^0}^{2r})^{\frac{1}{r}} ds\right)^r$ $t \in [0, b)$.

Definition 3.3. Let $p \geq 2$ be an integer. Eqn (2.2) is said to be stable in p-th moment if for arbitrarily given $\epsilon > 0$ there exist a $\delta > 0$ such that whenever $\|x_0\|_X < \delta$, $E\{\sup_{t\geq 0} \|x(t)\|_X^p\} < \epsilon$.

Definition 3.4. Let $p \geq 2$ be an integer. Eqn (2.2) is said to be asymptotically stable in p-th moment if it is stable in p-th moment and for any $x_0 \in X$, $\lim_{T\to\infty} E\{\sup_{t\geq T} \|x(t)\|_X^p\} = 0$.

Using the definitions and lemma given above, we consider the asymptotic stability in p-th moment of mild solutions of eqn (2.1) by using the contraction mapping principle. Imposing some Lipschitz and linear growth conditions on the function F and G, assume that $F(t, 0) = 0$, $G(t, 0) = 0$ and $I_k(0) = 0$ $(k = 1, 2, \dots)$. Then eqn (2.1) has a trival solution when $x_0 = 0$. Let X be the space of all Γ_0-adapted process $\phi(t, w) : [0, \infty) \times \Omega \to R$ which is almost certainly continuous in t for fixed $w \in \Omega$. Moreover, $\phi(0, w) = x_0$ and $E\|\Phi(t, w)\|_X^p \to 0$ as $t \to \infty$. Also X is a Banach space when it is equipped with a norm defined by

$$\|\phi\|_X = \sup_{t\geq 0} E\|\phi(t)\|_X^p.$$

We impose the following conditions:

1. A is the infinitesimal generator of a semigroup of bounded linear operators $S(t)$, $t \geq 0$ on a Banach space X with $\|S(t)\|_X \geq Me^{-at}$, $t \geq 0$ for some constants $M \geq 1$ and $0 < a \in R_+$.
2. The functions F and G satisfy the Lipschitz conditions and there exists a constant K for every $t \geq 0$ and $x, y \in X$ such that

$$\|F(t, x) - F(t, y)\|_X \leq K\|x - y\|_X,$$

$$\|G(t, x) - G(t, y)\|_X \leq K\|x - y\|_X.$$

3. $I_k \in C(X,X)$ and there exist a constant q_k such that $\|I_k(x)-I_k(y)\| \leq q_k\|x-y\|$ for each $x,y \in X$ $(k=1,\dots m)$.

Theorem 3.5. *Assume the conditions* (1–3) *hold. Let* $p \geq 2$ *be an integer. If the inequality* $3^{p-1}M^p(K^pa^{-p} + K^pc_p(2a)^{-p/2} + L) < 1$ *is satisfied, then the impulsive stochastic differential equation* (2.1) *is asymptotically stable in p-th moment: where* $c_p = (p(p-1)/2)^{p/2}$, $L = e^{-apT}E(\sum_{k=1}^{m}\|q_k\|_x^p)$.

Proof. Define a nonlinear operator $\psi : X \to X$ by

$$psix(t) = T(t)x_0 + \int_0^t T(t-s)F(s,x(s))ds + \int_0^t T(t-s)G(s,x(s))dW(s)$$
$$+ \sum_{0<t_k<t} T(t-t_k)I_k(x(t_k)) = \sum_{i=1}^{4} F_i(t)t \geq 0$$

To prove the asymptotic stability, it is enough to show that the operator ψ has a fixed point in X. To prove this result, we use the contraction mapping principle.

To apply the contraction mapping principle, we first verify the mean square continuity of ψ on $[0,\infty)$. Let $x \in X, t_1 \geq 0$ and $|r|$ be sufficiently small then

$$E\,\|\psi(x)(t_1+r) - \psi(x)(t_1)\|_X^p \leq 4^{p-1}\sum_{i=1}^{4} E\|F_i(t_1+r) - F_i(t_1)\|_X^p.$$

We see that $E\|F_i(t_1+r) - F_i(t_1)\|_X^p \to 0$, $i = 1,2,4$ as $r \to 0$. Moreover, by using Holder's inequality and Lemma 3.2, we obtain

$$E\|F_3(t_1+r) - F_3(t_1)\|_X^p \leq 2^{p-1}c_p$$
$$\times \left[\int_0^{t_1} (E\|(T(t_1+r-s) - T(t_1-s))G(s,x(s))\|_X^p)^{\frac{2}{p}}ds\right]^{(\frac{p}{2})}$$
$$+ 2^{p-1}c_p\left[\int_{t_1}^{t_1+r} (E\|(T(t_1+r-s)G(s,x(s))\|_X^p)^{\frac{2}{p}}ds\right]^{(\frac{p}{2})} \to 0$$

as $r \to 0$ where $c_p = (p(p-1)/2)^{\frac{p}{2}}$. Thus ψ is continuous in p-th moment on $[0,\infty)$.

Next we show that $\psi(X) \subset X$, and obtain

$$E\|(\psi x)(t)\|_X^p \leq 4^{p-1}E\|T(t)x_0\|_X^p$$
$$+ 4^{p-1}E\left\|\int_0^t T(t-s)F(s,x(s))ds\right\|_X^p$$
$$+ 4^{p-1}E\left\|\int_0^t T(t-s)G(s,x(s))dW(s)\right\|_X^p$$
$$+ 4^{p-1}\sum_{0<t_k<t} E\,\|T(t-t_k)I_k(x(t_k))\|_X^p.$$

Using conditions (1) and (3) we obtain $4^{p-1}E\left\|T(t)x_0\right\|_X^p \leq 4^{p-1}M^pe^{-pat}\|x_0\|_X^p \to 0$ as $t \to \infty$.

Now, from conditions (1) and (2) and Holder's inequality, we have

$$\begin{aligned}4^{p-1}E\left\|\int_0^t T(t-s)F(s,x(s))d(s)\right\|_X^p &\leq 4^{p-1}M^pk^p\left[\int e^{-a(t-s)}ds\right]^{p-1} \\ &\quad + \int e^{-a(t-s)}E\|x(s)\|_x^p ds \\ &\leq 4^{p-1}M^pK^pa^{1-p}\int_0^t e^{-a(t-s)}E\|x(s)\|_X^p ds.\end{aligned}$$

For any $x(t) \in X$ and any $\epsilon > 0$ there exist a $t_1 > 0$ such that $E\|x(s)\|_X^p < \epsilon$ for $t \geq t_1$. Thus we obtain

$$\begin{aligned}4^{p-1}E\left\|\int_0^t T(t-s)F(s,x(s))ds\right\|_x^p &\leq 4^{p-1}M^pK^pa^{1-p}e^{-at}\int_0^{t_1} e^{as}E\|x(s)\|_x^p ds \\ &\quad + 4^{p-1}M^pK^pa^{-p}\epsilon.\end{aligned}$$

As $e^{-at} \to 0$ as $t \to \infty$ and by assumption in Theorem 3.5, there exist $t_2 \geq t_1$ such that for any $t > t_2$ we have

$$4^{p-1}M^pK^pa^{1-p}e^{-at}\int_0^{t_1} e^{as}E\|x(s)\|_x^p ds \leq \epsilon - 4^{p-1}M^pK^pa^{-p}\epsilon.$$

we obtain for any $t \geq t_2$

$$4^{p-1}E\left\|\int_0^t T(t-s)F(s,x(s))ds\right\|_X^p < \epsilon$$

that is to say,

$$4^{p-1}E\left\|\int_0^t T(t-s)F(s,x(s))ds\right\|_X^p \to 0 \quad \text{as } t \to \infty.$$

Now for any $x(t) \in X$, $t \in [0,\infty)$, we obtain

$$\begin{aligned}&4^{p-1}E\left\|\int_0^t T(t-s)G(s,x(s))dW(s)\right\|_X^p \\ &\leq 4^{p-1}c_pM^pK^p\left[\int_0^t e^{-2a(t-s)}(E\|x(s)\|_X^p)^{2/p}ds\right]^{p/2}\end{aligned}$$

Further, we have

$$4^{p-1}E\left\|\int_0^t T(t-s)G(s,x(s))dW(s)\right\|_X^p \to 0 \quad \text{as } t \to \infty$$

$E\|(\psi x)(t)\|_X^p \to 0$ as $t \to \infty$. In conclusion, $\psi(X) \subset X$.

Finally, we prove that ψ is a contraction mapping. To see this, let $x, y \in X$, $s \in [0, T]$. Then,

$$\begin{aligned}
&\sup_{s\in[0,T]} E\|(\psi x)(t) - (\psi y)(t)\|_X^p \\
&\leq 3^{p-1} \sup_{s\in[0,T)} E\left\|\int_0^t T(t-s)(F(s,x(s)) - F(s,y(s)))ds\right\|_X^p \\
&\quad + 3^{p-1} \sup_{s\in[0,T]} E\left\|\int_0^t T(t-s)(G(s,x(s)) - G(s,y(s)))dW(s)\right\|_X^p \\
&\quad + 3^{p-1} \sup_{s\in[0,T]} E\left\|\sum_{0\leq t_k<t} T(t-t_k)(I_k(x(t_k)) - I_k(y(t_k))\right\|_X^p \\
&\leq \left[3^{p-1} M^p (K^p a^{-p} + K^p c_p (2a)^{p/2} + L)\right] \\
&\times \left(\sup_{s\in[0,T]} E\|x(t) - y(t)\|_X^p\right),
\end{aligned}$$

where $L = e^{-apT} E(\sum_{k=1}^m \|q_k\|_X^p)$. □

Therefore, ψ is a contraction mapping and hence there exist a unique fixed point $x(\cdot)$ in X which is the solution of eqn (2.1) with $x(0) = x_0$ and $E\|x(t)\|_x^p \to 0$ as $t \to \infty$.

REFERENCES

[1] A. Anokhin, L. Berezansky, E. Braverman, Exponential stability of linear delay impulsive differential equations, *Math. Anal. Appl.*, 193 (1995) 923–941.

[2] M. Benchohra, J. Henderson, and S. K. Ntouyas, *Impulsive differential equations and inclusions* Hindawi Publishing Corporation, vol 2., New York, 2006.

[3] P. Cheng, F. Q. Deng, Global exponential stability of impulsive stochastic functional differential systems, *Statist. Probab. Lett.*, 80 (2010) 1854–1862.

[4] M. M. Fu and Z. X. Liu, Square-mean almost automorphic solutions for some stochastic differential equations, *Pro. American Math. Soc.*, 138(10) (2010), 3689–3701.

[5] I. Karatzas, S. E. Shreve, *Brownian Motion and Stochastic Calculus*, Springer-Verlag, Berlin, 1991.

[6] V. Lakshmikantham, D. D. Bainov, and P. S. Simenov, *Theorem of Impulsive Differential Equations*, World Scientific, Singapore, 1989.

[7] B. Liu, X. Liu, K. Teo, Q. Wang, Razumikhin-type theorems on exponential stability of impulsive delay systems, *IMA J. Appl. Math*, 71 (2006) 47–61.

[8]] B. Liu, Stability of solutions for stochastic impulsive systems via comparison approach,*IEEE Trans. Automat. Contro*, 53 (2008) 2128–2133.

[9] X. Mao, *Stochastic Differential Equations and Applications*, Ellis Horwood, Chichester, UK, 1997.

[10] L. Pan, Existence of Mild Solutions for Impulsive Stochastic Differential Equations with Nonlocal Conditions, *Differential Equations and Application*, (2012), 485–494.

[11] S. G. Peng, B. G. Jia, Some criteria on pth moment stability of impulsive stochastic functional differential equations, *Statist. Probab. Lett.*, 80 (2010) 1085–1092.

[12] Q. Wang, X. Z. Liu, Impulsive stabilization of delay differential systems via the Lyapunov-Razumikhin method, *Appl. Math. Lett.*, 20 (8) (2007) 839–845.

[13] Q. J. Wu, J. Zhou, L. Xiang, Global exponential stability of impulsive differential equations with any time delays, *Appl. Math. Lett*, 23 (2010) 143–147.

[14] E. Zeidler, *Nonlinear Functional Anlysis and its Applications*, Springer-Verlag New York, (1986).