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THE EXISTENCE OF POSITIVE SOLUTIONS FOR A SEMIPOSITONE SECOND-ORDER m-POINT BOUNDARY VALUE PROBLEM

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ABSTRACT. In this paper, we study the existence of positive solutions to boundary value problem

$$\begin{cases} u'' + \lambda f(t,u) = 0, & t \in (0,1), \\ u(0) = \sum_{i=1}^{m-2} \alpha_i u(\xi_i), & u'(1) = \sum_{i=1}^{m-2} \beta_i u'(\xi_i), \end{cases}$$

where $\xi_i \in (0,1)$, $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$, $\alpha_i, \beta_i \in [0,\infty)$, λ is positive parameter. By using Krasnosel'skii's fixed point theorem, we provide sufficient conditions for the existence of at least one positive solution to the above boundary value problem.

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1. INTRODUCTION

The multipoint boundary value problems (BVPs) for ordinary differential equations arise in a variety of different areas of applied mathematics and physics.The study of multipoint BVPs for linear second-order ordinary differential equations was initiated by Il'in and Moiseev [9]. Since then there has been much current attention focused on the study of nonlinear multipoint BVPs; see [1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

Recently, Graef and Kong [6] studied the existence of positive solutions of the BVP

$$\begin{cases} u''' = \lambda f(t,u) + e(t), & t \in (0,1), \\ u(0) = u'(p) = \int_q^1 w(s) u'' ds = 0, \end{cases}$$

where $\lambda > 0$ is a parameter, $1/2 < p < q < 1$ are constants, $f : (0,1) \times [0,\infty) \to \mathbb{R}$, $e : (0,1) \to \mathbb{R}$, and $w : [q,1] \to [0,\infty)$ are continuous functions, and $e \in L(0,1)$. They found some sufficient conditions for the existence of positive solutions of a third order semipositone BVP with a multi-point boundary condition.

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Ma and Castaneda [14] investigated the existence of positive solutions for the BVP

$$\begin{cases} u'' + a(t)f(u) = 0, & t \in (0,1), \\ u'(0) = \sum_{i=1}^{m-2} a_i u'(\xi_i), & u(0) = \sum_{i=1}^{m-2} b_i u(\xi_i), \end{cases}$$

where $\xi_i \in (0,1)$, with $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$, $a_i, b_i \in [0,\infty)$. By using Krasnosel'skii's fixed point theorem in cones, they established the existence results for at least one positive solution to BVP, assuming that $0 < \sum_{i=1}^{m-2} a_i < 1$, $0 < \sum_{i=1}^{m-2} b_i < 1$, and $f \in C([0,\infty),[0,\infty))$, $a \in C([0,1),[0,\infty))$, where f is either superlinear or sublinear.

Kong and Kong [11] considered the boundary value problem with nonhomogeneous multi-point boundary condition

$$\begin{cases} u'' + a(t)f(u) = 0, & t \in (0,1), \\ u(0) = \sum_{i=1}^{m} a_i u(t_i) + \lambda, & u(1) = \sum_{i=1}^{m} b_i u(t_i) + \mu. \end{cases}$$

A sufficient condition is found for the existence and uniqueness of a positive solution. The dependence of the solution on the parameters λ and μ is also studied.

The present work is motivated by the recent paper [4, 6, 11], we intend in this paper to study the existence of at least one positive solution for second-order multipoint BVP

$$\begin{cases} u'' + \lambda f(t,u) = 0, & t \in (0,1), \\ u(0) = \sum_{i=1}^{m-2} \alpha_i u(\xi_i), & u'(1) = \sum_{i=1}^{m-2} \beta_i u'(\xi_i), \end{cases} \tag{1.1}$$

where $\xi_i \in (0,1)$, $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$, $\alpha_i, \beta_i \in [0,\infty)$, λ is positive parameter.

We study the semipositone BVP (1.1) and find sufficient conditions under which BVP (1.1) has a positive solution when $\lambda > 0$ is sufficiently small and large, respectively.

The following assumptions will stand throughout this paper:

(H1) $\alpha_i, \beta_i \in [0,\infty)$ for $0 < \sum_{i=1}^{m-2} \alpha_i < 1$ and $0 < \sum_{i=1}^{m-2} \beta_i < 1$;

(H2) $f : [0,1] \times [0,\infty) \to \mathbb{R}$ is continuous, and there exists $M > 0$, such that $f(t,u) \geq -M$ for $(t,u) \in [0,1] \times [0,\infty)$;

(H3) $f_\infty = \infty$;

(H4) $f^\infty = 0$;

(H5) there exist a nonnegative constant μ and a positive constant D such that $f(t,u) \geq \mu$, for $(t,u) \in [0,1] \times [D,\infty)$.

Set

$$f_\infty = \lim_{x\to\infty} \inf \min_{t\in[0,1]} \frac{f(t,x)}{x}, \qquad f^\infty = \lim_{x\to\infty} \sup \max_{t\in[0,1]} \frac{f(t,x)}{x}.$$

For boundary value problems, there is little literature that has referred to the existence of positive solutions when the nonlinearity can take a negative value. Inspired by the work [4, 6, 11, 14], in this paper, we consider the existence of positive solutions of the semipositone boundary value problem. Emphasis is put on the fact that the nonlinear term f may take a negative value. The results here are new, even in the cases of difference equations and differential equations.

By using Krasnosel'skii's fixed point theorem, we obtain positive solutions for a semipositone second-order multi-point boundary value problem. Compared to the results in [4, 6, 11, 14], our work presented in this paper has the following new features. Firstly, the conditions we used here differ from those in the majority of papers as we know. Secondly, the nonlinear term f may take a negative value.Thirdly, the existence of positive solutions obtained here includes not only for $\lambda > 0$ sufficiently small but also for $\lambda > 0$ sufficiently large.

Our main results extend and improve the main results of [4, 6, 11, 14]. Instead of the constant M by any continuous function $M(t)$ on $[0,1]$, the conclusions of Theorems 3.1 and 3.2 still hold.

The rest of paper is arranged as follows. In Section 2, we present some lemmas in order to prove our main results. In Section 3, we prove the existence of at least one positive solution for problem (1.1) by using Krasnosel'skii's fixed point theorem.

2. SOME LEMMAS

Let

$$E = C[0,1], \quad \gamma = \frac{1-\sum_{i=1}^{m-2}\alpha_i(1-\xi_i)}{\sum_{i=1}^{m-2}\alpha_i\xi_i}, \quad K = \{u : u\in C[0,1], \quad \min_{t\in[0,1]} u(t) \geq \gamma\|u\|\}.$$

Then E is a Banach space with norm $\|u\| = \max_{t\in[0,1]} \mid u(t) \mid$, and K is a cone in E.

Lemma 2.1. *If* $\left(1-\sum_{i=1}^{m-2}\alpha_i\right)\left(1-\sum_{i=1}^{m-2}\beta_i\right) \neq 0$*, then for* $h\in C[0,1]$ *and* $h \geq 0$*,*

$$\text{(2.1)} \qquad \begin{cases} u'' + h(t) = 0, & t\in(0,1), \\ u(0) = \displaystyle\sum_{i=1}^{m-2}\alpha_i u(\xi_i), & u'(1) = \sum_{i=1}^{m-2}\beta_i u'(\xi_i), \end{cases}$$

has the unique solution

(2.2)
$$\begin{cases} u(t) = -\int_0^t (t-s)h(s)ds + \dfrac{\left(\int_0^1 h(s)ds - \sum_{i=1}^{m-2}\beta_i \int_0^{\xi_i} h(s)ds\right)}{1-\sum_{i=1}^{m-2}\beta_i} t + \dfrac{1}{1-\sum_{i=1}^{m-2}\alpha_i} \\ \left[-\sum_{i=1}^{m-2}\alpha_i \int_0^{\xi_i} (\xi_i - s)h(s)ds + \dfrac{\left(\int_0^1 h(s)ds - \sum_{i=1}^{m-2}\beta_i \int_0^{\xi_i} h(s)ds\right)}{1-\sum_{i=1}^{m-2}\beta_i} \left(\sum_{i=1}^{m-2}\alpha_i\xi_i\right)\right]. \end{cases}$$

Moreover, if $h(t) \geq 0$ *on* $[0,1]$ *and* (H1) *is satisfied, then* $u(t) \geq 0$ *on* $[0,1]$.

Lemma 2.2. *Assume that (H1) holds. If* $h \in C[0,1]$ *and* $h \geq 0$*, then the unique solution* u *of BVP* (2.1) *satisfies*

$$\min_{t\in[0,1]} u(t) \geq \gamma\|u\|$$

where

$$\gamma = \frac{1-\sum_{i=1}^{m-2}\alpha_i(1-\xi_i)}{\sum_{i=1}^{m-2}\alpha_i\xi_i}.$$

Proof. Clearly $u'(t) \leq 0$. This implies that

$$\|u\| = u(0), \quad \min_{t\in[0,1]} u(t) = u(1).$$

It is easy to see that $u'(t_2) \leq u'(t_1)$ for any $t_1, t_2 \in [0,1]$ with $t_1 \leq t_2$. Hence $u'(t)$ is a decreasing function on $[0,1]$. This means that the graph of $u(t)$ is concave down on $(0,1)$. For each $i \in \{1,2,\ldots,m-2\}$, we have

$$\frac{u(\xi_i)-u(1)}{1-\xi_i} \geq \frac{u(0)-u(1)}{1},$$

i.e.,

$$-\xi_i u(1) \geq -u(\xi_i) + (1-\xi_i)u(0),$$

so that

$$-\sum_{i=1}^{m-2}\alpha_i\xi_i u(1) \geq -\sum_{i=1}^{m-2}\alpha_i u(\xi_i) + \sum_{i=1}^{m-2}\alpha_i(1-\xi_i)u(0),$$

and, with the boundary condition $u(0) = \sum_{i=1}^{m-2}\alpha_i u(\xi_i)$, we have

$$u(1) \geq \frac{1-\sum_{i=1}^{m-2}\alpha_i(1-\xi_i)}{\sum_{i=1}^{m-2}\alpha_i\xi_i} u(0).$$

This completes the proof. □

Lemma 2.3. *Assume that* (H1) *holds, then the problem*

$$\begin{cases} u'' + 1 = 0, & t \in (0,1), \\ u(0) = \sum_{i=1}^{m-2}\alpha_i u(\xi_i), & u'(1) = \sum_{i=1}^{m-2}\beta_i u'(\xi_i), \end{cases} \tag{2.3}$$

has the unique solution

$$\bar{v}(t) = \frac{-t^2}{2} + \frac{\left(1 - \sum_{i=1}^{m-2} \beta_i \xi_i\right)}{1 - \sum_{i=1}^{m-2} \beta_i} t$$
$$+ \frac{1}{1 - \sum_{i=1}^{m-2} \alpha_i} \left(-\frac{1}{2} \sum_{i=1}^{m-2} \alpha_i \xi_i^2 + \frac{\left(1 - \sum_{i=1}^{m-2} \beta_i \xi_i\right)}{1 - \sum_{i=1}^{m-2} \beta_i} \sum_{i=1}^{m-2} \alpha_i \xi_i \right)$$

and $\bar{v}(t) < C\gamma$. *Here*

$$C = \frac{1}{\gamma\left(1 - \sum_{i=1}^{m-2} \alpha_i\right)} \left(\frac{1}{1 - \sum_{i=1}^{m-2} \beta_i} \sum_{i=1}^{m-2} \alpha_i \xi_i \right), \quad t \in [0,1].$$

The proof of our main result based upon an application of the following fixed point theorem in a cone.

Theorem 2.4 ([8]). *Let E be a Banach space and let $K \subset E$ be a cone. Assume Ω_1, Ω_2 are open bounded subsets of E with $0 \in \Omega_1, \bar{\Omega}_1 \subset \Omega_2$ and let $F : K \cap (\bar{\Omega}_2 \backslash \Omega_1) \to K$ be a completely continuous operator such that*

(a) $\|Fu\| \leq \|u\|$, $u \in K \cap \partial\Omega_1$ *and* $\|Fu\| \geq \|u\|$, $u \in K \cap \partial\Omega_2$; *or*
(b) $\|Fu\| \geq \|u\|$, $u \in K \cap \partial\Omega_1$ *and* $\|Fu\| \leq \|u\|$, $u \in K \cap \partial\Omega_2$.

Then F has at least one fixed point in $K \cap (\bar{\Omega}_2 \backslash \Omega_1)$.

3. MAIN RESULTS

Theorem 3.1. *Assume that* (H1)–(H3) *hold, then BVP* (1.1) *has at least one positive solution for $\lambda > 0$ sufficiently small.*

Proof. Let $v(t) = \lambda M \bar{v}(t)$, then $u(t)$ is a positive solution of BVP (1.1) if $\bar{u}(t) = u(t) + v(t)$ is a solution of BVP

$$\begin{cases} u'' + \lambda g(t, u(t) - v(t)) = 0, & t \in (0,1), \\ u(0) = \sum_{i=1}^{m-2} \alpha_i u(\xi_i), & u'(1) = \sum_{i=1}^{m-2} \beta_i u'(\xi_i), \end{cases} \tag{3.1}$$

with $\bar{u}(t) > v(t)$, $t \in (0,1)$, where

$$g(t,u) = \begin{cases} f(t,u) + M, & (t,u) \in [0,1] \times [0,\infty), \\ f(t,0) + M, & (t,u) \in [0,1] \times [-\infty, 0). \end{cases}$$

We define the operator $F : K \to K$ by

(3.2)
$$\begin{cases} (Fu)(t) = -\lambda \displaystyle\int_0^t (t-s)g(s,u(s)-v(s))ds \\ +\dfrac{\lambda\Big(\int_0^1 g(s,u(s)-v(s))ds - \sum_{i=1}^{m-2}\beta_i\int_0^{\xi_i} g(s,u(s)-v(s))ds\Big)}{1-\sum_{i=1}^{m-2}\beta_i}t \\ +\dfrac{\lambda}{1-\sum_{i=1}^{m-2}\alpha_i}\Bigg[-\displaystyle\sum_{i=1}^{m-2}\alpha_i\int_0^{\xi_i}(\xi_i-s)g(s,u(s)-v(s))ds \\ +\dfrac{\Big(\int_0^1 g(s,u(s)-v(s))ds - \sum_{i=1}^{m-2}\beta_i\int_0^{\xi_i} g(s,u(s)-v(s))ds\Big)}{1-\sum_{i=1}^{m-2}\beta_i}\Big(\displaystyle\sum_{i=1}^{m-2}\alpha_i\xi_i\Big)\Bigg]. \end{cases}$$

It is easy to prove that the existence of solutions for BVP (3.1) is equivalent to the existence of solutions of the operator equation $Fu = u$. By Lemmas 2.1 and 2.2, this shows that $F(K) \subset K$. Moreover, it is easy to verify that F is completely continuous. Let

$$M_1 = \max_{0\le t,u\le 1} g(t,u), \quad \Lambda = \min\Big(\frac{1}{\gamma M_1 C}, \frac{1}{MC}\Big).$$

For $\lambda > 0$ sufficiently small such that $\lambda \in (0,\Lambda]$, choosing $u \in K$ with $\|u\| = 1$, we get

$$\begin{aligned} (Fu)(t) &\le \frac{\lambda}{1-\sum_{i=1}^{m-2}\alpha_i}\left[\frac{\int_0^1 g(s,u(s)-v(s))ds}{1-\sum_{i=1}^{m-2}\beta_i}\Big(\sum_{i=1}^{m-2}\alpha_i\xi_i\Big)\right] \\ &\le \frac{\lambda M_1}{1-\sum_{i=1}^{m-2}\alpha_i}\left[\frac{\int_0^1 ds}{1-\sum_{i=1}^{m-2}\beta_i}\Big(\sum_{i=1}^{m-2}\alpha_i\xi_i\Big)\right] \\ &= \frac{\lambda M_1}{1-\sum_{i=1}^{m-2}\alpha_i}\left[\frac{1}{1-\sum_{i=1}^{m-2}\beta_i}\Big(\sum_{i=1}^{m-2}\alpha_i\xi_i\Big)\right] = \lambda M_1\gamma C \le 1 = \|u\|. \end{aligned}$$

Thus, if we let $\Omega_1 = \{u \in K : \|u\| < 1\}$, then we have $\|Fu\| \le \|u\|$, for $u \in K \cap \partial\Omega_1$.

We choose a constant $N > 0$ such that $\frac{\gamma\lambda N}{2}\Big(1 - \frac{1}{2}\sum_{i=1}^{m-2}\beta_i\Big) \ge 1$. By the form of g and condition (H3), we know that $g(t,z)$ is an unbounded continuous function. Therefore, there exists $\bar{R} > 0$, such that $\Big(1 - \frac{\lambda CM}{\bar{R}}\Big) \ge \frac{1}{2}$ and

$$\frac{g(t,z)}{z} \ge N, \quad \text{for} \quad 0 \le t \le 1, \quad z(t) \ge \frac{1}{2}\gamma\bar{R}. \tag{3.3}$$

Let $R = \max(2,\bar{R})$ and $\Omega_R = \{u \in K : \|u\| < R\}$, for $u \in K$ with $\|u\| = R$, we get

$$v(t) = \lambda M\bar{v}(t) < \lambda MC\gamma \le \lambda MC\frac{u(t)}{\|u\|} = \frac{\lambda MC}{R}u(t) \le \frac{\lambda MC}{\bar{R}}u(t).$$

Since

$$u(t) - v(t) > \Big(1 - \frac{\lambda CM}{\bar{R}}\Big)u(t) \ge \frac{1}{2}u(t) \ge \frac{1}{2}\gamma\|u\| = \frac{1}{2}\gamma R \ge \frac{1}{2}\gamma\bar{R}. \tag{3.4}$$

From (3.3) and (3.4) , we find

$$g(t, u(t) - v(t)) \geq N(u(t) - v(t)) > \frac{1}{2}\gamma RN.$$

Hence, we obtain that

$$\begin{aligned}
(Fu)(0) &= \frac{\lambda}{1 - \sum_{i=1}^{m-2}\alpha_i}\Bigg[- \sum_{i=1}^{m-2}\alpha_i \int_0^{\xi_i}(\xi_i - s)g(s, u(s) - v(s))ds \\
&\quad + \frac{\left(\int_0^1 g(s, u(s) - v(s))ds - \sum_{i=1}^{m-2}\beta_i \int_0^{\xi_i}(\xi_i - s)g(s, u(s) - v(s))ds\right)}{1 - \sum_{i=1}^{m-2}\beta_i}\left(\sum_{i=1}^{m-2}\alpha_i\xi_i\right)\Bigg] \\
&\geq \lambda\left(\int_0^1 g(s, u(s) - v(s))ds - \sum_{i=1}^{m-2}\beta_i \int_0^1 (1 - s)g(s, u(s) - v(s))ds\right) \\
&\geq \frac{\lambda\gamma NR}{2}\left(\int_0^1 ds - \sum_{i=1}^{m-2}\beta_i \int_0^1 (1 - s)ds\right. \\
&= \frac{\lambda\gamma NR}{2}\left(1 - \frac{1}{2}\sum_{i=1}^{m-2}\beta_i\right) \geq R = \|u\|,
\end{aligned}$$

which implies that$\|Fu\| \geq \|u\|$, for $u \in K \cap \partial\Omega_R$.

It follows from Theorem 2.4 that F has a fixed point, therefore we can get BVP (3.1) has a least one positive solution $\bar{u}(t)$ with $1 \leq \|\bar{u}\| \leq R$. Combining with Lemma 2.2, we see that

$$\bar{u}(t) \geq \gamma\|\bar{u}\| \geq \lambda MC\gamma > \lambda M\bar{v}(t) = v(t), \quad t \in [0, 1].$$

Consequently, $u(t) = \bar{u}(t) - v(t)$ is a positive solution of BVP (1.1). □

Theorem 3.2. *Assume that* (H1), (H2), (H4) *and* (H5) *hold, then BVP* (1.1) *has at least one positive solution for* $\lambda > 0$ *sufficiently large.*

Proof. Let $R_1 = \max\left(\frac{2D}{\gamma}, 2\lambda MC\right)$, $\eta = R_1\left[(\mu + M)(1 - \frac{1}{2}\sum_{i=1}^{m-2}\beta_i)\right]^{-1}$. For $\lambda > 0$ sufficiently large such that $\lambda \in [\eta, \infty)$, choosing $u \in K$ with $\|u\| = R_1$, we get

$$v(t) = \lambda M\bar{v}(t) < \lambda MC\gamma \leq \lambda MC\frac{u(t)}{\|u\|} = \lambda MC\frac{u(t)}{R_1} \leq \frac{1}{2}u(t)$$

Since

$$u(t) - v(t) > \left(1 - \frac{\lambda CM}{R_1}\right)u(t) \geq \frac{1}{2}u(t) \geq \frac{1}{2}\gamma\|u\| = \frac{1}{2}\gamma R_1 \geq D, \tag{3.5}$$

and combining with the condition (H5), we find $g(t, u(t) - v(t)) = f(t, u(t) - v(t)) + M \geq \mu + M$.

Because of that, we can get that

$$(Fu)(0) = \frac{\lambda}{1 - \sum_{i=1}^{m-2}\alpha_i}\Bigg[- \sum_{i=1}^{m-2}\alpha_i \int_0^{\xi_i}(\xi_i - s)g(s, u(s) - v(s))ds$$

$$+\frac{\Big(\int_0^1 g(s,u(s)-v(s))ds-\sum_{i=1}^{m-2}\beta_i\int_0^{\xi_i}(\xi_i-s)g(s,u(s)-v(s))ds\Big)}{1-\sum_{i=1}^{m-2}\beta_i}\Big(\sum_{i=1}^{m-2}\alpha_i\xi_i\Big)\Big]$$

$$\geq \lambda\Big(\int_0^1 g(s,u(s)-v(s))ds-\sum_{i=1}^{m-2}\beta_i\int_0^1(1-s)g(s,u(s)-v(s))ds\Big)$$

$$\geq \lambda(\mu+M)\Big(1-\frac{1}{2}\sum_{i=1}^{m-2}\beta_i\Big)\geq R_1.$$

Thus, if we let $\Omega_{R_1}=\{u\in K:\|u\|<R_1\}$, then we have $\|Fu\|\geq\|u\|$, for $u\in K\cap\partial\Omega_{R_1}$.

From (H4), it is easy to see that

$$\lim_{x\to\infty}\sup\max_{t\in[0,1]}\frac{g(t,x)}{x}=\lim_{x\to\infty}\sup\max_{t\in[0,1]}\frac{f(t,x)+M}{x}=0.$$

Thus, let $L>0$ be chosen such that

$$g(t,x)\leq\epsilon x,\quad \text{for any}\quad 0\leq t\leq 1\quad\text{and}\quad x\geq L, \tag{3.6}$$

where $\epsilon>0$ with $\lambda C\gamma\epsilon\leq 1$. Let $R_2=\max\Big(2R_1,\frac{4L}{3\gamma}\Big)$ and $\Omega_{R_2}=\{u\in K:\|u\|<R_2\}$, for $u\in K$ with $\|u\|=R_2$, we get

$$v(t)=\lambda M\bar{v}(t)<\lambda MC\gamma\leq\lambda MC\frac{u(t)}{\|u\|}=\lambda MC\frac{u(t)}{R_2}\leq\frac{1}{4}u(t).$$

Since

$$u(t)-v(t)>\Big(1-\frac{\lambda CM}{R_2}\Big)u(t)\geq\frac{3}{4}u(t)\geq\frac{3}{4}\gamma\|u\|=\frac{3}{4}\gamma R_2\geq L, \tag{3.7}$$

from (3.6) and (3.7) , we find $g(t,u(t)-v(t))\leq\epsilon(u(t)-v(t))\leq\epsilon R_2$.

Therefore, it follows that

$$\begin{aligned}(Fu)(t)&\leq\frac{\lambda}{1-\sum_{i=1}^{m-2}\alpha_i}\Big[\frac{\int_0^1 g(s,u(s)-v(s))ds}{1-\sum_{i=1}^{m-2}\beta_i}\Big(\sum_{i=1}^{m-2}\alpha_i\xi_i\Big)\Big]\\&\leq\frac{\lambda\epsilon R_2}{1-\sum_{i=1}^{m-2}\alpha_i}\Big[\frac{1}{1-\sum_{i=1}^{m-2}\beta_i}\Big(\sum_{i=1}^{m-2}\alpha_i\xi_i\Big)\Big]\\&=\lambda C\gamma\epsilon R_2\leq R_2.\end{aligned}$$

This implies that $\|Fu\|\leq\|u\|$, for $u\in K\cap\partial\Omega_{R_2}$.

It follows from Theorem 2.4 that F has a fixed point, we can get that BVP (3.1) has a least one positive solution $\bar{u}(t)$ with $R_1\leq\|\bar{u}\|\leq R_2$. Combining with Lemma 2.2, we see that

$$\bar{u}(t)\geq\gamma\|\bar{u}\|\geq\gamma R_1\geq 2\lambda MC\gamma>\lambda M\bar{v}(t)=v(t),\quad t\in[0,1].$$

Consequently, $u(t)=\bar{u}(t)-v(t)$ is a positive solution of BVP (1.1). □

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