Dynamic Systems and Applications 24 (2015) 295-304

ON THE EXISTENCE OF POSITIVE SOLUTIONS FOR THE ONE-DIMENSIONAL p-LAPLACIAN BOUNDARY VALUE PROBLEMS ON TIME SCALES

ABDULKADIR DOGAN

Department of Applied Mathematics,Faculty of Computer Sciences, Abdullah Gul University, Kayseri, 38039 Turkey

ABSTRACT. In this paper, we study the following p-Laplacian boundary value problems on time scales

$$\begin{cases} (\phi_p(u^\Delta(t)))^\nabla + a(t)f(t,u(t),u^\Delta(t)) = 0, & t \in [0,T]_{\mathbb{T}}, \\ u(0) - B_0(u^\Delta(0)) = 0, & u^\Delta(T) = 0, \end{cases}$$

where $\phi_p(u) = |u|^{p-2}u$, for $p > 1$. We prove the existence of triple positive solutions for the one-dimensional p-Laplacian boundary value problem by using the Leggett-Williams fixed point theorem. The interesting point in this paper is that the non-linear term f is involved with first-order derivative explicitly. An example is also given to illustrate the main result.

AMS (MOS) Subject Classification. 34B15, 34B16, 34B18, 39A10.

1. INTRODUCTION

Recently, dynamic equations on time scales have generated a considerable amount of interest and attracted many researchers. They can not only unify differential and difference equations[11], but also have exhibited much more complicated dynamics [5]. Further they have led to several important applications e.g., in the study of insect population models, stock market, wound healing, and epidemic models [6, 13, 16].

In this paper, by using different method, we will discuss the existence of at least three positive solutions to the following p-Laplacian boundary value problem on time scales

$$\begin{cases} (\phi_p(u^\Delta(t)))^\nabla + a(t)f(t,u(t),u^\Delta(t)) = 0, & t \in [0,T]_{\mathbb{T}}, \\ u(0) - B_0(u^\Delta(0)) = 0, & u^\Delta(T) = 0, \end{cases} \tag{1.1}$$

where $\phi_p(u)$ is p-Laplacian operator, i.e., $\phi_p(u) = |u|^{p-2}u$, for $p > 1$, with $(\phi_p)^{-1} = \phi_q$ and $\frac{1}{p} + \frac{1}{q} = 1$. The usual notation and terminology for time scales as can be found in [5, 6], will be used here.

Agarwal and O'Regan [2] studied the existence of one or more solutions to nonlinear equations on time scales. They established by using either a nonlinear alternative of Leray-Schauder type or Krasnoselski's fixed point theorem in a cone. Dogan et

Received April 2, 2015 1056-2176 $15.00 ©Dynamic Publishers, Inc.

al. [7] considered some existence criteria for positive solutions of a higher order semi-positone multi-point boundary value problem on a time scale. We also discussed applications to some special problems. He and Jiang [10] investigated the existence of at least three positive solutions of boundary value problems for p-Laplacian dynamic equations on time scales by applying a new triple fixed-point theorem. Hong [12] presented sufficient conditions for the existence of at least three positive solutions of three-point boundary value problems for p-Laplacian dynamic equations on a time scales. To show his main results, he applied a new fixed point theorem due to Avery and Peterson. Su et al. [19] considered the three-point boundary value problem for p-Laplacian dynamic equations on time scales. They proved that the boundary value problem has at least three positive pseudo-symmetric solutions under some assumptions by using a pseudo-symmetric technique and the five-functionals fixed point theorem.

In recent years, there have been many papers working on the existence of positive solutions for p-Laplacian boundary value problems for differential equations on time scales, see, for example [3, 10, 12, 15, 17, 18, 19, 20, 21]. However, to best of our knowledge, there are not much concerning the p-Laplacian boundary value problems on time scales when the nonlinear term f is involved with the first-order delta derivative [8, 9].

We assume the following conditions hold through the paper:

(H1) $f \in C_{ld}((0,T) \times \mathbb{R}^2, (0,\infty)), \quad 0, T \in \mathbb{T}$,

(H2) $a \in C_{ld}((0,T),(0,\infty))$, $\min_{t\in[0,T]_{\mathbb{T}}} a(t) = \phi_p(m)$, $\max_{t\in[0,T]_{\mathbb{T}}} a(t) = \phi_p(M)$ and $m < M$,

(H3) B_0 is continuous function defined on $\mathbb{R}$ and satisfies that there exist $A \geq 1$ and $B > 0$ such that $Bv \leq B_0(v) \leq Av$, for all $v \in [0,+\infty)$.

2. PRELIMINARIES

Let $E = C^1_{ld}[0,T]$ with the norm

$$\|u\| = \max\{\|u\|_0, \|u^\Delta\|_0\},$$

where $\|u\|_0 = \sup\limits_{t\in[0,T]_{\mathbb{T}}} |u(t)|$; clearly E is Banach space. Choose the cone $P \subset E$ defined by

$$P = \Big\{u \in E : u(t) \geq 0, \text{ for } t \in [0,T]_{\mathbb{T}}; u^{\Delta\nabla}(t) \leq 0, \ u^\Delta(t) \geq 0, \text{ for } t \in [0,T]_{\mathbb{T}}\Big\}.$$

We note that $u(t)$ is a solution of (1.1), if and only if

$$\begin{cases} u(t) = \displaystyle\int_0^t \phi_q\left(\int_s^T a(r) f(r,u(r),u^\Delta(r))\nabla r\right)\Delta s \\ +B_0\left(\phi_q\left(\displaystyle\int_0^T a(r) f(r,u(r),u^\Delta(r))\nabla r\right)\right), \quad t \in [0,T]_{\mathbb{T}} \end{cases}$$

Define a completely continuous integral operator $F : P \to E$

$$\begin{cases} (Fu)(t) = \int_0^t \phi_q \left(\int_s^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \Delta s \\ +B_0 \left(\phi_q \left(\int_0^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \right), \quad u \in P \text{ for } t \in [0, T]_{\mathbb{T}}. \end{cases}$$

Clearly, $\|Fu\| = \max\{(Fu)(0), |(Fu)^\Delta(T)|\} = T_0(Fu)(0)$, where $T_0 = \max\{T, 1\}$.

Lemma 2.1. $FP \subset P$

Proof. In fact

$$(Fu)^\Delta(t) = \phi_q \left(\int_t^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \geq 0.$$

Moreover, $\phi_q(x)$ is a monotone decreasing and continuously differential function and

$$\left(\int_t^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right)^\nabla = -a(t) f(r, u(t), u^\Delta(t)) \leq 0,$$

we have $(Fu)^{\Delta\nabla}(t) \leq 0$, therefore $FP \subset P$. □

Lemma 2.2. $F : P \to P$ *is completely continuous.*

Proof. Firstly, we will show that F maps a bounded set into itself. Suppose $c > 0$ is a constant and $u \in \bar{P}_c = \{u \in P : \|u\| \leq c\}$, and then $|u| \leq c$, $|v| \leq c$; notice that $f(t, u, v)$ is continuous, therefore there exist a constant $C > 0$ such that $f(t, u, v) \leq \phi_p(C)$, and hence

$$\begin{aligned} \|Fu\| &= T_0(Fu)(0) \\ &= T_0 B_0 \left(\phi_q \left(\int_0^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \right) \\ &\leq T_0 A C \phi_q \left(\int_0^T a(r) \nabla r \right). \end{aligned}$$

That is $F\bar{P}$ is uniformly bounded. On the other hand,

$$\begin{aligned} |(Fu)(t_1) - (Fu)(t_2)| &= \left| \int_0^{t_1} \phi_q \left(\int_s^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \Delta s \right. \\ &\quad \left. - \int_0^{t_2} \phi_q \left(\int_s^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \Delta s \right| \\ &\leq \left| \int_{t_1}^{t_2} \phi_q \left(\int_0^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \Delta s \right| \\ &\leq C |t_1 - t_2| \phi_q \left(\int_0^T a(r) \nabla r \right), \end{aligned}$$

therefore F is equicontinuous on $[0, T]_{\mathbb{T}}$; then by applying the Arzela-Ascoli theorem on time scales [1], we know that $F\bar{P}$ is relatively compact. Using Lebesque's dominated convergence theorem on time scales [4], F is completely continuous on $[0,T]_{\mathbb{T}}$. $\square$

Let $a, b, r > 0$ be constants, $P_r = \{u \in P : \|u\| < r\}$, $P(\alpha, a, b) = \{u \in P : \alpha(u) \geq a, \|u\| < b\}$.

To prove our main results, we need the following Leggett-William fixed point Theorem [14].

Theorem 2.3 (Leggett-Williams)**.** *Let $F : \bar{P}_c \to \bar{P}_c$ be a completely continuous map and α be a nonnegative continuous concave functional on P such that $\alpha(u) \leq \|u\|$, $\forall u \in \bar{P}_c$. Assume there exist a, b, d with $0 < a < b < d \leq c$ such that*

(A1) $\{u \in P(\alpha, b, d) : \alpha(u) > b\} \neq \emptyset$, *and* $\alpha(Fu) > b$ *for all* $u \in P(\alpha, b, d)A$;
(A2) $\|Fu\| < a$, *for all* $u \in \bar{P}_a$;
(A3) $\alpha(Fu) > b$ *for all* $u \in P(\alpha, b, c)$ *with* $\|Fu\| > d$.

Then F has at least three fixed points u_1, u_2, u_3 satisfying

$$\|u_1\| < a, \qquad b < \alpha(u_2), \qquad \|u_3\| > a, \qquad \alpha(u_3) < b.$$

3. MAIN RESULTS

We define the nonnegative continuous concave functional $\alpha : P \to [0, \infty)$ by

$$\alpha(u) = \min_{t \in [l, T-l]} u(t), \qquad l = \max\{t \in T : t \in [0, T/2]\},$$

$$K = \left(\frac{l+B}{l}\right) \phi_q \left(\int_0^{T-l} a(r) \nabla r\right).$$

Clearly, the following two conclusions hold:

(i) $\alpha(u) = u(l) \leq \|u\|$, $\forall u \in P$;
(ii) $\alpha(Fu) = u(l)$.

Theorem 3.1. *Assume that there exist constants a, b, c, d such that $0 < a < b \leq \frac{(l+B)m}{MBT_0} d < d \leq c$ and suppose that f satisfies the following conditions:*

(B1) $f(t, u, v) \leq \phi_p\left(\frac{aT^{\frac{1}{1-p}}}{MT_0}\right)$ *for* $(t, u, v) \in [0, T]_{\mathbb{T}} \times [0, a] \times [-a, 0]$;
(B2) $f(t, u, v) \leq \phi_p\left(\frac{cT^{\frac{1}{1-p}}}{MT_0}\right)$ *for* $(t, u, v) \in [0, T]_{\mathbb{T}} \times [0, c] \times [-c, 0]$;
(B3) $f(t, u, v) > \phi_p\left(\frac{b}{lK}\right)$ *for* $(t, u, v) \in [0, T-l]_{\mathbb{T}} \times [b, d] \times [-d, 0]$;
(B4) $\min\{f(t, u, v)\} \phi_p\left(\frac{M}{m}\right) \int_0^{T-l} a(t) \nabla t \geq \max\{f(t, u, v)\} \int_0^T a(t) \nabla t$ *for* $(t, u, v) \in [0, T]_{\mathbb{T}} \times [0, c] \times [-c, 0]$.

Then the boundary value problem (1.1) *has at least three positive solutions u_1, u_2 and u_3 satisfying*

$$\|u_1\| < a, \quad b < \alpha(u_2), \quad \|u_3\| > a, \quad \alpha(u_3) < b.$$

Proof. First, we show that there exists a positive number $c > d$ such that $F\bar{P}_c \subset \bar{P}_c$, $F\bar{P}_a \subset \bar{P}_a$. From Lemma 2.1 $F\bar{P}_c \subset \bar{P}_c$, and then $\forall u \in \bar{P}_c$, from B2, we have $0 \le u \le c$, $-c \le v \le 0$,

$$\begin{aligned} \|Fu\|_0 &\le T\phi_q\Big(\int_0^T a(r)f(r,u(r),u^\Delta(r))\nabla r\Big) \\ &\quad +A\Big(\phi_q\Big(\int_0^T a(r)f(r,u(r),u^\Delta(r))\nabla r\Big)\Big) \\ &= (T+A)\phi_q\Big(\int_0^T a(r)f(r,u(r),u^\Delta(r))\nabla r\Big) \\ &\le (T+A)\phi_q\Big(\phi_p(M)\phi_p\Big(\frac{c}{T_0M}\Big)\Big) \le c \\ \|(Fu)^\Delta\|_0 &\le \phi_q\Big(\phi_p(M)\phi_p\Big(\frac{c}{T_0M}\Big)\Big) \le c. \end{aligned}$$

Similarly, $Fu \in \bar{P}_a$ for all $u \in \bar{P}_a$.

Second, we show $\{u \in P(\alpha,b,d) : \alpha(u) > b\} \ne \emptyset$, and $\alpha(Fu) > b$ for all $u \in P(\alpha,b,d)$. In fact, set $u = \frac{b+d}{2}$, $\|u\| = \frac{b+d}{2} \le d$ and $\alpha(u) > b$. Therefore $\{u \in P(\alpha,b,d) : \alpha(u) > b\} \ne \emptyset$. On the other hand, $\forall u \in P(\alpha,b,d)$, we get $b \le u \le d$, $-d \le v \le 0$, and for $t \in [0,T-l]_{\mathbb{T}}$, from B3,

$$\begin{aligned} \alpha(Fu) &= \int_0^l phi_q\Big(\int_s^T a(r)f(r,u(r),u^\Delta(r))\nabla r\Big)\Delta s \\ &\quad +B_0\Big(\phi_q\Big(\int_0^T a(r)f(r,u(r),u^\Delta(r))\nabla r\Big)\Big) \\ &\ge \int_0^l \phi_q\Big(\int_0^{T-l} a(r)f(r,u(r),u^\Delta(r))\nabla r\Big)\Delta s \\ &\quad +B\Big(\phi_q\Big(\int_0^{T-l} a(r)f(r,u(r),u^\Delta(r))\nabla r\Big)\Big) \\ &= (l+B)\phi_q\Big(\int_0^{T-l} a(r)f(r,u(r),u^\Delta(r))\nabla r\Big) \\ &= (l+B)\frac{b}{lK}\phi_q\Big(\int_0^{T-l} a(r)\nabla r\Big) = b. \end{aligned}$$

Hence $\alpha(Fu) > b$ for $u \in P(\alpha,b,d)$.

Finally, we show $\alpha(Fu) > b$ for all $u \in P(\alpha,b,d)$ and $\|Fu\| > d$. If $u \in P(\alpha,b,d)$ and $\|Fu\| > d$, then $0 \le u \le c$, $-c \le v \le 0$, and from B4,

$$\phi_p\Big(\frac{M}{m}\Big)\int_0^{T-l} a(r)f(r,u(r),u^\Delta(r))\nabla r \ge \int_0^T a(r)f(r,u(r),u^\Delta(r))\nabla r,$$

i.e.

$$\int_0^{T-l} a(r)f(r,u(r),u^\Delta(r))\nabla r \ge \frac{\int_0^T a(r)f(r,u(r),u^\Delta(r))\nabla r}{\phi_p\Big(\frac{M}{m}\Big)}.$$

Because $Fu \in P$,

$$\begin{aligned}
\alpha(Fu) &= (Fu)(l) \\
&= \int_0^l \phi_q\left(\int_s^T a(r)f(r,u(r),u^{\Delta}(r))\nabla r\right)\Delta s \\
&\quad +B_0\left(\phi_q\left(\int_0^T a(r)f(r,u(r),u^{\Delta}(r))\nabla r\right)\right) \\
&\geq l\phi_q\left(\int_0^{T-l} a(r)f(r,u(r),u^{\Delta}(r))\nabla r\right) \\
&\quad +B\left(\phi_q\left(\int_0^{T-l} a(r)f(r,u(r),u^{\Delta}(r))\nabla r\right)\right) \\
&= (l+B)\phi_q\left(\int_0^{T-l} a(r)f(r,u(r),u^{\Delta}(r))\nabla r\right) \\
&\geq (l+B)\phi_q\left(\frac{\int_0^T a(r)f(r,u(r),u^{\Delta}(r))\nabla r}{\phi_p\left(\frac{M}{m}\right)}\right) \\
&= \frac{(l+B)m}{M}\phi_q\left(\int_0^T a(r)f(r,u(r),u^{\Delta}(r))\nabla r\right) \\
&\geq \frac{(l+B)m}{M}\frac{Fu(0)}{B} \\
&= \left(\frac{l+B}{M}\right)m\frac{\|Fu\|}{BT_0} \\
&\geq \left(\frac{l+B}{M}\right)m\frac{d}{BT_0} \geq b,
\end{aligned}$$

and then $\alpha(Fu) > b$ for all $u \in P(\alpha,b,d)$ and $\|Fu\| > d$. Hence, an application of Theorem 2.3 completes the proof. □

Theorem 3.2. *Assume that there exist constants a,b,c,d such that $0 < a < b \leq \frac{(l+B)m}{MBT_0}d < d \leq c$ and suppose that f satisfies* (B1)–(B3) *and*

(C1) *$a(t)$ is decreasing for $t \in [0,T]_{\mathbb{T}}$,*
(C2) $\phi_p\left(\frac{M}{m}\right) \geq \frac{(T-l)f_M}{lf_m}$,

where

$$f_M = \max\{f(t,u,v)\} \text{ for } (t,u,v) \in [l,T]_{\mathbb{T}} \times [0,c] \times [-c,0],$$

$$f_m = \min\{f(t,u,v)\} \text{ for } (t,u,v) \in [0,l]_{\mathbb{T}} \times [0,c] \times [-c,0].$$

Then the boundary value problem (1.1) *has at least three positive solutions u_1, u_2 and u_3 satisfying*

$$\|u_1\| < a, \quad b < \alpha(u_2), \quad \|u_3\| > a, \quad \alpha(u_3) < b.$$

Proof. We only show $\alpha(Fu) > b$ for all $u \in P(\alpha, b, c)$ and $\|Fu\| > d$. $\forall u \in P(\alpha, b, c)$, we have $0 \le u \le c$, $-c \le v \le 0$, and from (C1), we get

$$\int_0^l a(l)f(r, u(r), u^{\Delta}(r))\nabla r \le \int_0^l a(r)f(r, u(r), u^{\Delta}(r))\nabla r,$$
$$\int_l^T a(l)f(r, u(r), u^{\Delta}(r))\nabla r \ge \int_l^T a(r)f(r, u(r), u^{\Delta}(r))\nabla r,$$

and then

$$\begin{aligned}\frac{\int_l^T a(r)f(r, u(r), u^{\Delta}(r))\nabla r}{\int_0^l a(r)f(r, u(r), u^{\Delta}(r))\nabla r} &\le \frac{\int_l^T a(l)f(r, u(r), u^{\Delta}(r))\nabla r}{\int_0^l a(l)f(r, u(r), u^{\Delta}(r))\nabla r} \\ &= \frac{\int_l^T f(r, u(r), u^{\Delta}(r))\nabla r}{\int_0^l f(r, u(r), u^{\Delta}(r))\nabla r} \\ &\le \frac{(T-l)f_M}{lf_m}.\end{aligned}$$

From (C2), we have

$$\phi_p\Big(\frac{M}{m}\Big) \ge \frac{\int_l^T a(r)f(r, u(r), u^{\Delta}(r))\nabla r}{\int_0^l a(r)f(r, u(r), u^{\Delta}(r))\nabla r},$$

and the rest of the proof is similar to final step of Theorem 3.1 so we omit it. □

4. EXAMPLE

Let $\mathbb{T} = \mathbb{R}$, $T = 1$, $p = 3$, then $T_0 = 1$. Consider the following boundary value problem

$$\begin{cases} |u'|u' + a(t)f(t, u(t), u'(t)) = 0, & t \in [0, 1], \\ u(0) - \frac{1}{2}u'(0) = 0, \quad u'(1) = 0, \end{cases} \tag{4.1}$$

where

$$a(t) = \begin{cases} 1, & t \in [0, \frac{14}{25}] \\ -t + \frac{4000001}{4000000}, & t \in [\frac{14}{25}, 1], \end{cases}$$

$$f(t, u, v) = \begin{cases} 10u^5 + \frac{2+\sin v}{180}, & \text{for } 0 \le t \le 1,\ 0 \le u \le 1,\ -\frac{\pi}{2} \le v \le 0; \\ 10u^5 + \frac{1}{180}, & \text{for } 0 \le t \le 1,\ 0 \le u \le 1,\ v \le -\frac{\pi}{2}; \\ 10\sqrt[20]{u} + \frac{2+\sin v}{180}, & \text{for } 0 \le t \le 1,\ u \ge 1,\ -\frac{\pi}{2} \le v \le 0; \\ 10\sqrt[20]{u} + \frac{1}{180}, & \text{for } 0 \le t \le 1,\ u \ge 1,\ v \le -\frac{\pi}{2}. \end{cases}$$

By the definition of $a(t)$, we know $m = \frac{1}{2000}$ and $M = 1$. It is obvious that $A = B = \frac{1}{2}$. Choose $l = \frac{11}{25}$, a direct calculation shows that

$$K = \left(\frac{l+B}{l}\right)\phi_q\left(\int_0^{T-l} a(r)dr\right) = \frac{(\frac{11}{25} + \frac{1}{2})}{\frac{11}{25}}\sqrt{\frac{14}{25}} = \frac{47}{22}\sqrt{\frac{14}{25}}.$$

If we take $a = \frac{1}{2}$, $b = 1$, $d = 1250$, $c = 1260$, then

$$0 < \frac{1}{2} < 1 < \frac{(\frac{11}{25} + \frac{1}{2}) \times \frac{1}{2000}}{\frac{1}{2}} \times 1250 < 1250 < 1260,$$

we have

$$\phi_3\Big(\frac{a}{M}\Big) = \Big(\frac{1}{2}\Big)^2 = \frac{1}{4},\ \phi_3\Big(\frac{c}{M}\Big) = 1260^2,\ \phi_3\Big(\frac{b}{lK}\Big) = \frac{25 \times 22^2 \times 25^2}{11^2 \times 47^2 \times 14},$$

then the nonlinear term f satisfies

(B1) $f(t,u,v) \leq 10\times\Big(\frac{1}{2}\Big)^5 + \frac{1}{90} < \frac{1}{4} = \phi_3\Big(\frac{a}{M}\Big),$

$$\text{for } 0 \leq t \leq 1,\ 0 \leq u \leq \frac{1}{2},\ -\frac{1}{2} \leq v \leq 0;$$

(B2) $f(t,u,v) \leq 10 + \frac{1}{60} < 1260^2 = \phi_3\Big(\frac{c}{M}\Big),$

$$\text{for } 0 \leq t \leq 1,\ 0 \leq u \leq 1,\ -1260 \leq v \leq 0;$$

$$f(t,u,v) \leq 10\sqrt[20]{1260} + \frac{1}{60} < 1260^2 = \phi_3\Big(\frac{c}{M}\Big),$$

$$\text{for } 0 \leq t \leq 1,\ 1 \leq u \leq 1260,\ -1260 \leq v \leq 0;$$

(B3) $f(t,u,v) \geq 10 + \frac{1}{180} > \frac{25 \times 22^2 \times 25^2}{11^2 \times 47^2 \times 14} = \frac{31250}{19963} = \phi_3\Big(\frac{b}{lK}\Big)$

$$\text{for } 0 \leq t \leq \frac{14}{25},\ 1 \leq u \leq 1250,\ -1250 \leq v \leq 0;$$

(B4) $\min\{f(t,u,v)\} = \frac{1}{180}$ for $0 \leq t \leq 1,\ 0 \leq u \leq 1260,\ -1260 \leq v \leq 0;$

$$\max\{f(t,u,v)\} = 10\sqrt[20]{1260} + \frac{1}{60} \text{ for } 0 \leq t \leq 1,\ 0 \leq u \leq 1260,\ -1260 \leq v \leq 0.$$

$$\phi_3\Big(\frac{M}{m}\Big) = 2000^2,\ \int_0^{1-l} a(t)dt = \frac{14}{25},$$

$$\int_0^{\frac{14}{25}} 1dt + \int_{\frac{14}{25}}^1 \Big(-t + \frac{4000001}{4000000}\Big)dt = \frac{14}{25} + \frac{9680011}{100000000} = \frac{65680011}{100000000}.$$

We have

$$\min\{f(t,u,v)\}\phi_3\Big(\frac{M}{m}\Big)\int_0^{1-l} a(t)dt \geq \max\{f(t,u,v)\}\int_0^1 a(t)dt$$

$$\text{for } (t,u,v) \in [0,1] \times [0,1260] \times [-1260,0].$$

$$\frac{1}{180} \times 2000^2 \times \frac{14}{25} > \Big(10\sqrt[20]{1260} + \frac{1}{60}\Big) \times \Big(\frac{65680011}{100000000}\Big).$$

Thus by Theorem 3.1, we find that boundary value problem (4.1) has at least three positive solutions.

Acknowledgments. The author would like to thank the anoymous referees and editor for their helpful comments and suggestions.

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