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OPTIMAL DIVIDEND PROBLEM WITH THE INFLUENCE OF DIVIDEND PAYOUTS ON INSURANCE BUSINESS

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ABSTRACT. This article initiates the optimal dividend problem, from the view point of the managers of the insurance companies. where we incorporate the influence of dividend payouts on the insurance business. We begin with a mathematical characterization of the influence of dividend payouts, and then continue to find the optimal dividend policy that maximizes the expected utility of terminal wealth and minimizes the ruin probability. We study the problem in terms of the Levy process and derive the diffusion process case as a particular one.

Keywords: Dividend Payouts; HJB equation; Ruin Probability

AMS Classification: 93E20, 60H30, 60H10

1. INTRODUCTION

The optimal dividend payout is a classical problem in actuarial mathematics. This problem was first proposed by de Finetti (see, [6]) in order to cope with the unrealistic problem of minimizing ruin probability. Since then, the optimal dividend problem has been addressed in numerous papers due to its practical importance. This optimal dividend payout analysis has turned out to be a rich and challenging field of research.

The optimal dividend problem has been studied in the setting of Cramér-Lundberg model [4], diffusion process model [14], and negative Lévy Process model [3]. Given a dividend policy, the performance measure includes the expected value of discounted future dividend payments [6], the expected discounted utility of a dividend stream [11], and the expected utility of the cumulative discounted dividend payments [9]. We also note that several other controls have been included; for example, proportional

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reinsurance, excess of loss reinsurance, investment, tax and transaction cost, and interdependent claims. The detailed extensions of the optimal dividend problem can be found in the survey articles: Albrecher and Thonhauser [1], Avanzi [2], Hipp [10], Schmidli [15], Taksar [16], and references therein.

An important assumption in recent research is that the dividend payout has no influence on insurance business. However, this assumption is far from reality in the insurance business. One of the trend in this field is to incorporate more general and more realistic model assumptions. In addition to the transaction cost, the influence of dividend payouts on insurance business is an important factor in dividend policy-making. This situation which needs to be, but has not been, considered in the research of optimal dividend problem.

There are two parties to the dividends payments in the modern insurance corporations. First of all, there are the *insiders*, namely, the managers of the firm, and then there are the *outsiders*, the shareholders or policyholder. Indeed, the interests of the insiders and of the outsiders may not always coincide. This has important consequences for a dividend policy. There is a suggestion that the insider typically prefers a low payout in order to pursue the company's growth or consume additional benefits. However, the outsider generally wish for a high payout since this will force the management to incur the inspection of the capital markets for each new project undertaken. Most of the research up until now concentrated on the viewpoint of the outsiders; in other words, the goal is to maximize the expected value of dividend payments. In this paper, we study the optimal dividend problem with the objectives of minimizing the ruin probability and maximizing the expected utility of terminal wealth, which measure the safety and profit of the insurance company, respectively.

The rest of the paper is organized as follows. In Section 2, we give a mathematical characterization of the influence of dividend payouts on the insurance business. In Section 3, we solve the optimal dividend problem with two different value functions: minimizing the ruin probability and maximizing the expected utility of terminal wealth. In section 4, we state some extensions of the model.

2. **THE EFFECT OF DIVIDEND PAYMENTS**

A theory suggests that the company announcements of increases in dividend payouts act as an indicator of the firm possessing strong future prospects. Dividends can give investors a sense of what a company is really worth. The rationale behind dividend influence models stems from game theory. In insurance business, the dividend policy can influence the growth renewal premiums. If a company that has a history of steady or consistently increasing dividend payments suddenly cuts its payments, then the investors should treat this as a signal that trouble is looming. In insurance market, the sudden cut of dividend payments will result in a lose of the renewal

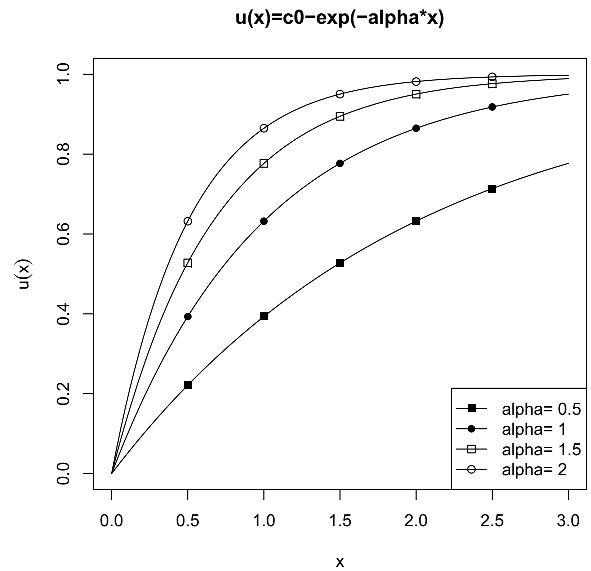

FIGURE 1. $u(x)$ for different α

premiums. Otherwise, the growth in renewal premiums will continue. Many cases in practice have shown that dividend signaling does occur when companies either increase or decrease the amount of dividends they will be paying out.

Incorporating the influence of dividend payouts on insurance business in the optimal dividend problem necessitates one to provide a mathematical characterization of the influence of dividend payouts.

According to the above discussion, the dependent and independent variables of the influence function should be premium income and dividend payouts, respectively. The influence function should be monotone, bounded and positive. As an example, we take the following form of the influence function, which satisfies the above requirements:

$$\mu(x) = c_0 - \exp(-\alpha x), \tag{1}$$

where $c_0 > 1, \alpha > 0$ are constants.

The parameter α measures the extent of the influence of dividend payouts on the premiums. As we can see from the Figure 1, the influence of dividend payout on the premiums becomes greater as the parameter α becomes larger.

3. THE MODEL AND THE SOLUTION

In the absense of dividend payouts, the dynamics of the surplus of the insurance company can be modeled as

$$dR(t) = \mu dt + \sigma dW_t - dS(t). \tag{2}$$

Here: $\mu > 0$ is the constant premium income rate; $S(t) = \sum_{i=1}^{N(t)} Y_i$ is a compound Poisson process that represents the cumulative claims up to time t, i.e., the jumps sizes Y_i are i.i.d. with distribution Φ, and (N_t) is a Poisson process with intensity λ, independent of $(Y_i)_{i\geq 1}$; and $\{W_t : t \geq 0\}$ is a standard Brownian motion with σdW_t representing the additional uncertainty associated with the insurance market or the economic environment.

Let L_t denote the aggregate of dividends by time t. We say that L_t ia an admissible control if it is an $\mathcal{F}_t-$ adapted, monotonically increasing, positive process. Let Π be the set of all admissible controls. We assume through out this paper that L_t has a Radon-Nikodym derivative l_s w.r.t the Lebesgue measure:

$$L_t = \int_0^t l_s ds, \quad 0 \leq l_s \leq M$$

where M is the maximum rate of dividend payout. Taking into account the influence of dividends on the insurance business, the insurance company's surplus $X^l(t)$ can be formulated as follows:

$$dX^l(t) = (\mu(l_t) - l_t)dt + \sigma dW_t - dS(t) \tag{3}$$

where $X^l(0) = x$ and recall $\mu(x) = c_0 - \exp(-\alpha x)$.

3.1. **Maximizing the expected utility of terminal wealth.** Assuming that the insurer's objective is to maximize the exponential utility of the terminal wealth, say at terminal time T, the value function has the following form:

$$V(t,x) = \sup_{l\in\Pi} V_l(x) = \sup_{l\in\Pi} E[u(X_T^l)|X_t^l = x]. \tag{4}$$

where, $u(x)$ is the exponential utility function:

$$u(x) = c_0 - \frac{\delta}{\gamma} e^{-\gamma x}, \tag{5}$$

where $\delta, \gamma > 0$, $u'(x) > 0, u''(x) < 0$.

Applying Itô formula to an $f(t,x) \in C^{(1,2)}$, [12], we obtain the generator of $X^l(t)$ governed by the Equation (3) as

$$\mathcal{A}^l f(t,x) = f_t + [c_0 - \exp(-\alpha l) - l]f_x + \frac{1}{2}\sigma^2 f_{xx} + \lambda \mathbb{E}[V(t, x-Y) - V(t,x)],$$

where, f_x, f_t and f_{xx} denote the first order partial derivative with respect to x, the first order partial derivative with respect to t, and the second order partial derivative with respect to x, respectively, Y is a random variable with distribution Φ.

Assuming that the value function V is smooth enough, we appeal to dynamic programming [8], to notice that the value function V satisfies the Hamilton-Jacobi-Bellman equation as presented in the following theorem.

Theorem 1. *If the value function V defined by (4) is twice continuously differentiable on $(0,\infty)$, then V satisfies the following equation*

$$\sup_{0\le l\le M}\{V_t+(c_0-\exp(-\alpha l)-l)V_x+\frac{1}{2}\sigma^2V_{xx}+\lambda\mathbb{E}[V(t,x-Y)-V(t,x)]\}=0 \tag{6}$$

with the terminal condition

$$V(T,x)=u(x).$$

Because V is increasing (and hence the first order derivative V_x is positive), the supremum in Equation (6) is equivalent to

$$\sup\,[c_0-\exp(-\alpha l)-l]\ \text{ on } [0,M].$$

Indeed, the derivative of the function under the supremum is $g(l):=\alpha\exp(-\alpha l)-1$.

Case 1. Let $0<\alpha\le 1$. Here, $g(l)<0$, for all $l\in[0,M]$. So the maximizer of the left hand side of Equation (6) is $l^*=0$. Thus the HJB equation (6) simplifies to

$$V_t+(c_0-1)V_x+\frac{1}{2}\sigma^2V_{xx}+\lambda\mathbb{E}[V(t,x-Y)-V(t,x)]=0. \tag{7}$$

To solve Equation (7), we follow Browne [5] and fit a solution of the form

$$V(t,x)=c_0-\frac{\delta}{\gamma}\exp[-\gamma x+h(T-t)], \tag{8}$$

where $h(\cdot)$ is a suitable function so that (8) is a solution of (7). The terminal condition $V(T,x)=u(x)$ implies that $h(0)=0$. For this trial solution, we have

$$V_t(t,x)=[V(t,x)-c_0]\cdot[-h'(T-t)], \tag{9}$$

$$V_x(t,x)=[V(t,x)-c_0]\cdot[-\gamma], \tag{10}$$

$$V_{xx}(t,x)=[V(t,x)-c_0]\cdot[\gamma^2]. \tag{11}$$

$$\begin{aligned}\mathbb{E}[V(t,x-Y)-V(t,x)]&=[V(t,x)-c_0]\left[\int_0^\infty \exp(\gamma y)d\Phi-1\right]\\&=[V(t,x)-c_0]\mathbb{E}[\exp(\gamma Y)-1].\end{aligned} \tag{12}$$

Inserting (9)–(12) into (7), we obtain

$$h'(T-t)=-(c_0-1)\gamma+\frac{1}{2}\sigma^2\gamma^2+\mathbb{E}[\exp(\gamma Y)-1].$$

$$h(T-t)=\left[-(c_0-1)\gamma+\frac{1}{2}\sigma^2\gamma^2+\mathbb{E}[\exp(\gamma Y)-1]\right](T-t).$$

Since the conditions of the verification theorem in [7] are easily verified in our case, the above discussion gives the following theorem.

Theorem 2. *If $\alpha\le 1$, then the value function is*

$$V(x)=c_0-\frac{\delta}{\gamma}\exp\Big(-\gamma x+(-(c_0-1)\gamma+\frac{1}{2}\sigma^2\gamma^2+\mathbb{E}[\exp(\gamma Y)-1])(T-t)\Big).$$

and the optimal dividend policy is $l^*=0$.

Case 2. **Let** $\alpha > 1$. Similar to the case of minimizing the ruin probability (that will be considered below), we shall consider the following two subcases:

i) **Let** $M < \frac{1}{\alpha}\log(\alpha)$:

In this case, the optimizer of Equation (6) is $l^* = M$. The HJB Equation (6) reduces to

$$V_t + (c_0 - \exp(-\alpha M) - M)V_x + \frac{1}{2}\sigma^2 V_{xx} + \lambda\mathbb{E}[V(t, x - Y) - V(t, x)] = 0.$$

The solution of this equation is:

$$V(x) = c_0 - \frac{\delta}{\gamma}\exp\Big(- \gamma x + (-(c_0 - \exp(-\alpha M) - M)\gamma + \frac{1}{2}\sigma^2\gamma^2 + \mathbb{E}[\exp(\gamma Y) - 1])(T - t)\Big).$$

ii) **Let** $M > \frac{1}{\alpha}\log(\alpha)$:

In this case, the optimizer of Equation (6) is

$$l^* = \frac{1}{\alpha}\log(\alpha),$$

and the HJB equation (6) simplifies to

$$V_t + (c_0 - \frac{1}{\alpha} - \frac{1}{\alpha}\log(\alpha))V_x + \frac{1}{2}\sigma^2 V_{xx} + \lambda\mathbb{E}[V(t, x - Y) - V(t, x)] = 0.$$

The solution of the above equation is

$$V(x) = c_0 - \frac{\delta}{\gamma}\exp\Big(- \gamma x + (-(c_0 - \frac{1}{\alpha} - \frac{1}{\alpha}\log(\alpha))\gamma + \frac{1}{2}\sigma^2\gamma^2 + \mathbb{E}[\exp(\gamma Y) - 1])(T - t)\Big).$$

The above discussions along with the verification theorem in [7] give the following theorem.

Theorem 3. *If $\alpha > 1$, then the value function and the optimal dividend policy have the following form:*

a) *In the case of $M < \frac{1}{\alpha}\log(\alpha)$, the value function is*

$$V(x) = c_0 - \frac{\delta}{\gamma}\exp\Big(- \gamma x + (-(c_0 - \exp(-\alpha M) - M)\gamma + \frac{1}{2}\sigma^2\gamma^2 + \mathbb{E}[\exp(\gamma Y) - 1])(T - t)\Big).$$

and the optimal dividend policy is $l^* = M$.

b) *the case of $M > \frac{1}{\alpha}\log(\alpha)$ the value function is*

$$V(x) = c_0 - \frac{\delta}{\gamma}\exp\Big(- \gamma x + (-(c_0 - \frac{1}{\alpha} - \frac{1}{\alpha}\log(\alpha))\gamma + \frac{1}{2}\sigma^2\gamma^2 + \mathbb{E}[\exp(\gamma Y) - 1])(T - t)\Big).$$

and the optimal dividend policy is $l^* = \frac{1}{\alpha}\log(\alpha)$.

3.2. **Minimizing the Ruin Probability.** We consider, in this subsection, the optimal problem to minimize the ruin probability. The *ruin time* is defined by:

$$\tau_l := \inf\{t > 0 : \ X^l(t) \leq 0\} \tag{13}$$

We define the *ruin probability* $V_l(x)$ when the initial surplus is x and the rate of dividend payout is l; that is,

$$V_l(x) := P_x(\tau_l < \infty) = P(\tau_l < \infty | X^l(0) = x)$$

Our goal is to find the optimal policy that minimizes the ruin probability:

$$V(x) := \inf_{l \in \Pi} V_l(x) = \inf_{l \in \Pi} P(\tau_l < \infty | X^l(0) = x) \tag{14}$$

such that

$$V_{l^*}(x) = V(x).$$

Applying Itô formula to a C^2 function $f(x)$ [12], we obtain the generator of X^l :

$$\mathcal{A}^l f(x) = \frac{1}{2}\sigma^2 f_{xx} + [c_0 - \exp(-\alpha l) - l] f_x + \lambda \mathbb{E}[V(x - Y) - V(x)].$$

Appealing once again to the theory of dynamic programming [8], we see that when the value function is smooth enough, it satisfies the following Hamilton-Jacobi-Bellman equation.

Theorem 4. *Assume that V defined by* (14) *is twice continuously differentiable on* $(0, \infty)$. *Then V satisfies the following equation:*

$$\inf_{0 \leq l \leq M} \left\{ \frac{1}{2}\sigma^2 V_{xx} + (c_0 - \exp(-\alpha l) - l) V_x \right\} = 0 \tag{15}$$

with the boundary conditions

$$V(0) = 1,$$
$$V(\infty) = 0.$$

Because V is decreasing and the first order derivative V_x is negative, the infimum in equation (15) is equivalent to

$$\sup\, [c_0 - \exp(-\alpha l) - l] \ \text{ on } [0, M],$$

where the derivative of the function under the supremum is $g(l) = \alpha \exp(-\alpha l) - 1$. In this case, the solution of HJB Equation (15) can also be discussed in the following three subcases:

- $\alpha \leq 1$
- $\alpha > 1, \ M < \frac{1}{\alpha} \log(\alpha)$
- $\alpha > 1, \ M > \frac{1}{\alpha} \log(\alpha)$

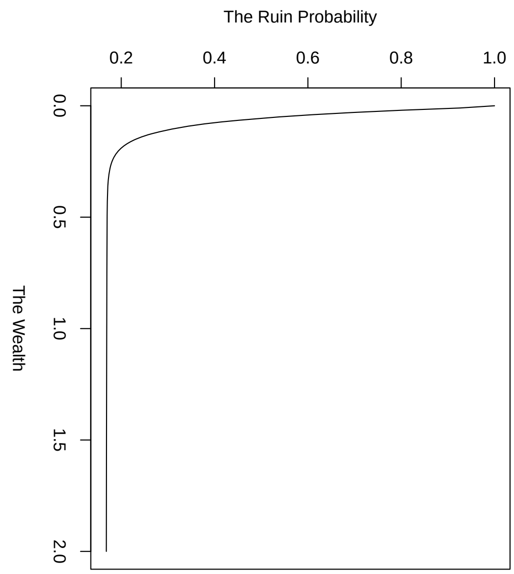

FIGURE 2. $u(x)$ for different α

Without lose of generality, we solve the problem for the first case: $\alpha \leq 1$, and the other two cases can be solved similarly. In this case, we get $g(l) < 0$ for all $l \in [0, M]$. So the maximizer of the left hand side of Equation (15) is $l^* = 0$. The HJB equation (15) simplifies to

$$(c_0 - 1)V_x + \frac{1}{2}\sigma^2 V_{xx} + \lambda \mathbb{E}[V(x - Y) - V(x)] = 0 \tag{16}$$

Consider the survival function $F(y) = 1 - \Phi(y)$, where $\Phi(y)$ is the distribution function of the claim-size Y. Then the equation (16) can be rewritten as

$$(c_0 - 1)V_x + \frac{1}{2}\sigma^2 V_{xx} - \lambda \int_0^x V_x(x - y)F(y)dy - \lambda V(0)F(x) = 0. \tag{17}$$

Let $g(x) = V_x$, then

$$(c_0 - 1)g(x) + \frac{1}{2}\sigma^2 g'(x) - \lambda \int_0^x g(x - y)F(y)dy - \lambda V(0)F(x) = 0 \tag{18}$$

The Equation (18) is a Volterra type integro-differential equation of the second kind. We cannot solve it analytically and hence we proceed with the "guess and verify" technique. Toward this, we adopt the method to solve the Volterra integral equation of the second kind as presented in [13].

As we can see in Figure 2, the ruin probability is decreasing as the wealth increases. Also, the ruin probability has similar shape with the ruin probability in the jump-diffusion model.

3.3. **The Optimal Problem with Diffusion Risk Process.** In actuarial science, the diffusion process has been popular in describing the risk process. In several problems problems, the case of diffusion model is easier to deal with than the case of jump-diffusion model. It's also true that in the diffusion case the optimal dividend problem of either minimizing the ruin probability or maximizing the terminal wealth can be solved analytically. The diffusion risk process without dividend payments is as follows:

$$dR(t) = \mu dt + \sigma dW_t, \tag{19}$$

where $\mu > 0$ is the constant premium income rate. When the dividends are paid out, the dynamics of insurance company's surplus $X^l(t)$ can be formulated as follows:

$$dX^l(t) = (\mu(l_t) - l_t)dt + \sigma dW_t \tag{20}$$

where $X^l(0) = x$ and $\mu(x) = c_0 - \exp(-\alpha x)$.

As in the case of jump-diffusion risk model studied above, we consider two value functions, namely (a) minimizing the ruin probability and (b) maximizing the terminal wealth. These two value functions both can be derived though the "guess and verify" method. First, we consider the optimal policy that minimizes the ruin probability:

$$V(x) = \inf_{l\in\Pi} V_l(x) = \inf_{l\in\Pi} P(\tau_l < \infty | X^l(0) = x), \tag{21}$$

where the ruin time τ_l is defined as in (13).

By solving the corresponding HJB equation, we get the value function and the optimal dividend policy. We summarize the standard diffusion case from the jump diffusion case in the following theorem.

Theorem 5. 1) *If $\alpha \leq 1$, then the value function has the form:*

$$V(x) = \exp(-\frac{2(c_0 - 1)}{\sigma^2} x),$$

and the optimal dividend policy is $l^* = 0$.

2) *If $\alpha > 1$, then the value function and the optimal dividend policy are given as follows:*

a) *In the case of $M < \frac{1}{\alpha}\log(\alpha)$, the value function is*

$$V(x) = \exp(-\frac{2(c_0 - \exp(-\alpha M) - M)}{\sigma^2} x),$$

and the optimal dividend policy is $l^* = M$.

b) *In the case of $M > \frac{1}{\alpha}\log(\alpha)$, the value function is*

$$V(x) = \exp(-\frac{2(c_0 - \frac{1}{\alpha} - \frac{1}{\alpha}\log(\alpha))}{\sigma^2} x),$$

and the optimal policy is $l^* = \frac{1}{\alpha}\log(\alpha)$.

We shall consider next the optimal policy that maximizes the terminal wealth:

$$V(t,x)=\sup_{l\in\Pi}V_l(x)=\sup_{l\in\Pi}E[u(X_T^l)|X_t^l=x]. \tag{22}$$

where the utility function is defined as in (5).

We get the associated value function and the optimal dividend policy by solving the corresponding HJB equation. We have the following theorem.

Theorem 6. 1) *If $\alpha\le 1$, then the value function is*

$$V(x)=c_0-\frac{\delta}{\gamma}\exp\left(-\gamma x+(-(c_0-1)\gamma+\frac{1}{2}\sigma^2\gamma^2)(T-t)\right)$$

and the optimal dividend policy is $l^*=0$.

2) *If $\alpha>1$, then the value function and the optimal dividend policy are given by:*

a) *In the case of $M<\frac{1}{\alpha}\log(\alpha)$, the value function is*

$$V(x)=c_0-\frac{\delta}{\gamma}\exp\left(-\gamma x+(-(c_0-\exp(-\alpha M)-M)\gamma+\frac{1}{2}\sigma^2\gamma^2)(T-t)\right)$$

and the optimal dividend policy is $l^*=M$.

b) *In the case of $M>\frac{1}{\alpha}\log(\alpha)$ the value function is*

$$V(x)=c_0-\frac{\delta}{\gamma}\exp\left(-\gamma x+(-(c_0-\frac{1}{\alpha}-\frac{1}{\alpha}\log(\alpha))\gamma+\frac{1}{2}\sigma^2\gamma^2)(T-t)\right)$$

and the optimal dividend policy is $l^*=\frac{1}{\alpha}\log(\alpha)$.

Remark 1. We get the same optimal dividend policy to minimize the ruin probability and maximize the exponential utility of the terminal wealth. The optimal dividend policy can be stated as follows:

1) If $\alpha\le 1$,
   $l^*=0$
2) If $\alpha>1$,
   a) $l^*=M$, provided $M<\frac{1}{\alpha}\log(\alpha)$, and
   b) $l^*=\frac{1}{\alpha}\log(\alpha)$, provided $M>\frac{1}{\alpha}\log(\alpha)$.

From the above results, we can see that when the dividend payout has little influence on the insurance business (the case of $\alpha\le 1$), obviously the optimal dividend policy is to have no dividend payout. The managers of insurance company typically prefer a low payout in order to pursue company's future growth or consume additional benefits. In the case of low cost to cut dividend payments, the optimal choice is no dividend payout again. But when the dividend payout has appreciable influence on the insurance business (the case of $\alpha>1$), the situation is different. In this case the insurance company may not be able to afford the loss of premium because of the low dividend payment. So the dividend payout is maintained at a relatively high level. Those are in accordance with what most insurance companies are doing in practice.

As we can see, the rate of dividend payout does not tend to infinity as the maximum rate of dividend payout M tends to infinity. The finiteness of the optimal policy is partly due to the boundedness of dividend influence function $u(\cdot)$ and this fits with realitye, which in turn supports the necessity of the boundedness of $u(\cdot)$.

Remark 2. *Why do we need to consider the influence of dividend on insurance business*?

We answer this by solving the optimal dividend problem without the influence of dividend on insurance business. Without the influence of dividends on the insurance business, the insurance company's surplus $X^l(t)$ can be formulated as follows:

$$dX^l(t) = (\mu - l_t)dt + \sigma dW_t$$

The value function is the minimization of the ruin probability:

$$V(x) = \inf_{l\in\Pi} V_l(x) = \inf_{l\in\Pi} P(\tau_l < \infty | X^l(0) = x)$$

We can easily prove that the value function satisfies the following equation:

$$\inf_{0\leq l\leq M} \left\{ \frac{1}{2}\sigma^2 V_{xx} + (\mu - l)V_x \right\} = 0.$$

It's not difficult to see that the optimal dividend policy of $l^* = 0$ is not realistic in practice. The no-dividend-payout policy is not always the best choice for the company's sake. Sometimes the managers may sacrifice their own profit in order to pay dividend. The reason that we get an unrealistic optimal policy here is that we do not consider the influence of dividend on the insurance business.

4. Concluding Remarks

In this paper, we study the influence of dividend payouts on the insurance business in terms of optimal dividend problem. However, we did not consider other controls in the model, such as the investment and the reinsurance. In our future work, we shall incorporate the controls such as the investment and the reinsurance that make the mathematical model more realistic.

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